

## 第 18 回広島仙台整数論集会アブストラクト

2019 年 7 月 9 日 (火) ~ 7 月 12 日 (金)  
広島大学理学部 (東広島キャンパス) B 棟 B707 号室

### 7 月 9 日 (火)

10:45 – 11:45 佐藤 信夫 (九州大学) / Nobuo Sato (Kyushu University)

*A multivariable generalization of Zagier's identity for  $\zeta(2, \dots, 2, 3, 2, \dots, 2)$  and Zhao's 2-1 type formula*

Zhao showed a beautiful mysterious identity which relates the block structure of multiple zeta star values and the depth structure of Euler sums. Zagier proved a formula expressing block degree one Hoffman basis  $\zeta(2, \dots, 2, 3, 2, \dots, 2)$  by a  $\mathbb{Q}$ -linear combination of  $\zeta(2l+1)\pi^{2k}$ . Zagier's formula can be reformulated and generalized into a simple iterated integral identity which has striking similarity to Zhao's formula. In my talk, I'll introduce a new type of differential forms and generalize those identities to multivariable iterated integral identities. This is a joint research work with Minoru Hirose.

12:00 – 13:00 広瀬 稔 (九州大学) / Minoru Hirose (Kyushu University)

*Alternating multiple zeta values and their bases*

Alternating multiple zeta values are signed analogues of multiple zeta values sometimes called Euler sums, mod two multiple zeta values, etc.. In his work on motivic multiple  $L$ -values, Deligne gave an explicit basis for the space of motivic alternating multiple zeta values. His proof makes use of the theory of mixed Tate motives, which does not provide a way to compute the expression (coefficients) of a given alternating multiple zeta values by his basis. The explicit reduction of alternating multiple zeta values into linear combinations of Deligne's basis (or any other explicit basis) is an open problem. In this talk, I report some recent progress on this problem made in a collaboration with Nobuo Sato at Kyushu University.

14:30 – 15:30 長町 一平 (東京大学) / Ippei Nagamachi (University of Tokyo)

*On the Hom version of the Grothendieck Conjecture for morphisms from regular varieties to hyperbolic polycurves of dimension 2*

Mochizuki proved the Hom version of the Grothendieck Conjecture for hyperbolic curves, that is, a conjecture that the map

$$\mathrm{Mor}_K^{\mathrm{dom}}(Y, X) \rightarrow \mathrm{Hom}_{G_K}^{\mathrm{open}}(\pi_1(Y, *), \pi_1(X, *))/\mathrm{Inn}$$

is bijective for any sub- $p$ -adic field  $K$ , any hyperbolic curve  $X$  over  $K$ , and any smooth variety  $Y$  over  $K$ . In the case where  $X$  is a hyperbolic polycurve of dimension 2, Hoshi proved the injectivity of the map and that any element  $\phi \in \mathrm{Hom}_{G_K}^{\mathrm{open}}(\pi_1(Y, *), \pi_1(X, *))/\mathrm{Inn}$  with topologically finitely generated kernel is in the image of the map. In this talk, we give a "group theoretic" condition for  $\phi$  such that  $\phi$  satisfies this condition if and only if  $\phi$  is in the image of the map. We also show that any open outer homomorphism is induced by a morphism of the varieties under the assumption that the Grothendieck Section Conjecture holds.

15:45 – 16:45 笠浦 一海 (東京大学) / Kazumi Kasaura (University of Tokyo)

*On extension of overconvergent log isocrystals on log smooth varieties*

By works of Kedlaya and Shiho, it is known that, for a smooth variety  $\overline{X}$  over a field of positive characteristic and its simple normal crossing divisor  $Z$ , an overconvergent isocrystal on the complement of  $Z$  satisfying a certain monodromy condition can be extended to a convergent log isocrystal on  $(\overline{X}, \mathcal{M}_Z)$ , where  $\mathcal{M}_Z$  is the log structure associated to  $Z$ . In this talk, we explain a generalization of this result: for a log smooth variety  $(\overline{X}, \mathcal{M})$  satisfying some conditions, an overconvergent log isocrystal on the trivial locus of a direct summand of  $\mathcal{M}$  satisfying a certain monodromy condition can be extended to a convergent log isocrystal on  $(\overline{X}, \mathcal{M})$ .

17:00 – 18:00 高田 芽味 (九州産業大学) / Megumi Takata (Kyushu Sangyo University)

*$p$  進体  $F$  の過収束性と  $F$  解析性との関係について*

*On a relation of overconvergence and  $F$ -analyticity on  $p$ -adic Galois representations of a  $p$ -adic field  $F$*

Let  $p$  be a prime number. There are a property called “overconvergence” for  $p$ -adic Galois representations of a  $p$ -adic field  $F$ . If a given  $p$ -adic Galois representation of  $F$  is overconvergent, then we can study it by using the theory of  $(\varphi, \Gamma)$ -modules over the univariable Robba ring. The case  $F = \mathbb{Q}_p$  is nice, since any  $p$ -adic representation is overconvergent, which is proved by Cherbonnier-Colmez. Unfortunately, if  $F \neq \mathbb{Q}_p$ , then there is a  $p$ -adic Galois representation of  $F$  which is not overconvergent. Berger has proved that another property called “ $F$ -analyticity” yields overconvergence. We note that  $F$ -analyticity is very strong, as one can see that the  $p$ -adic cyclotomic character is not  $F$ -analytic if  $F \neq \mathbb{Q}_p$ . Hence it seems to be a natural question that how strict  $F$ -analyticity is in comparison to overconvergence. In this talk, I discuss a result that, in many cases, an overconvergent Galois representation is  $F$ -analytic up to a twist by a character. This result emphasizes the necessity of the theory of  $(\varphi, \Gamma)$ -modules over the multivariable Robba ring, by which we expect to study all  $p$ -adic Galois representations.

## 7月10日(水)

9:30 – 10:30 奥村 喜晶 (東京工業大学) / Yoshiaki Okumura (Tokyo Institute of Technology)

*Parametrization of  $Q$ -virtual Drinfeld modules*

Elliptic curves over  $\mathbb{Q}$  are classical and important objects in number theory. As a generalization, it is natural to consider elliptic curves over  $\overline{\mathbb{Q}}$  which are isogenous to all  $G_{\mathbb{Q}}$ -conjugates, which are called  $\mathbb{Q}$ -curves. It is known that various properties of elliptic curves over  $\mathbb{Q}$  can be extended to  $\mathbb{Q}$ -curves. For example, Elkies proved that all isogeny classes of non-CM  $\mathbb{Q}$ -curves are corresponds to  $\mathbb{Q}$ -rational points of some modular curves.

Let  $Q = \mathbb{F}_q(T)$  be a rational function field over a finite field  $\mathbb{F}_q$ . In the arithmetic of function fields, Drinfeld modules (of rank two) over  $Q$  play the role that elliptic curves over  $\mathbb{Q}$  do in the arithmetic of number fields. In this talk, we introduce the notion of  $Q$ -virtual Drinfeld modules as a function field analogue of  $\mathbb{Q}$ -curves, and to parametrize (up to isogeny) them by  $Q$ -rational points of the Drinfeld modular curves  $X_*(\mathfrak{n})$ .

10:45 – 11:45 関川 隆太郎 (東京理科大学) / Ryutaro Sekigawa (Tokyo University of Science)

*Relative power integral bases in ray class fields of an imaginary quadratic number field*

Let  $L/K$  be an extension of number fields and let  $\mathcal{O}_L$  and  $\mathcal{O}_K$  be the rings of integers of  $L$  and  $K$ , respectively. It is said that  $\mathcal{O}_L$  has a  $\mathcal{O}_K$ -power integral basis when there is  $\alpha \in \mathcal{O}_L$  such that  $\mathcal{O}_L = \mathcal{O}_K[\alpha]$ . Let  $K$  be an imaginary quadratic number field and  $H$  be the Hilbert class field of  $K$ . Schertz proved that ray class fields of  $K$  have a  $\mathcal{O}_H$ -P.I.B. with some exceptions. Let  $K = \mathbb{Q}(\sqrt{-10})$  and  $\mathfrak{p} \subset K$  be the prime ideal over 2. In this talk, I explain the way to construct a P.I.B. for  $K_{\mathfrak{p}^4}/H$  that is one of the exceptions and introduce its application. The main tools of this way are fundamental units by PARI/GP.

12:00 – 13:00 田嶋 和明 (仙台高専・東北大学) /

Kazuaki Tajima (Sendai National College of Technology, Tohoku University)

*On the GIT stratification of prehomogeneous vector spaces*

We have established a combinatorial method to determine a stratification (GIT stratification) based on geometric invariant theory. We applied this method to certain prehomogeneous vector spaces using computer computations and determined their GIT stratifications. The result has been known if the ground field is  $\mathbb{C}$ . This method enables us to determine the stratification rationally over any perfect field. This is joint work with Akihiko Yuki.

14:30 – 15:30 福永 健吾 (大阪大学) / Kengo Fukunaga (Osaka University)

*$p$ -adic triple product  $L$ -function attached to  $p$ -adic families of modular forms*

In his preprint “Hida families and  $p$ -adic triple product  $L$ -functions”, Ming-Lun Hsieh constructed three-variable  $p$ -adic triple product  $L$ -functions attached to triples  $(F, G, H)$  of primitive Hida families and he proved the interpolation formulas.

In the unbalanced case, I generalized his result and constructed a three-variable  $p$ -adic triple product  $L$ -function attached to a primitive Hida family  $F$  and general  $p$ -adic families of modular forms  $G, H$ . For example, we can take  $G$  and  $H$  to be Colman families or CM-families. I will explain the construction of this  $p$ -adic triple product  $L$ -function.

15:45 – 16:45 伊藤 望 (京都大学) / Nozomi Ito (Kyoto University)

*On Miyawaki lifts with respect to Hermitian Maass lifts*

Miyawaki lifts are liftings constructed by using the pullbacks of Ikeda lifts. They have been studied intensively in recent years, but their properties, especially non-vanishing, have not been sufficiently understood. In this talk, we study Miyawaki lifts of automorphic characters of  $U(1)$  with respect to Hermitian Maass lifts, which is Ikeda lifts for  $U(2, 2)$ .

7月11日(木)

9:30 – 10:30 石本 宙 (京都大学) / Hiroshi Ishimoto (Kyoto University)

*Local intertwining relation for metaplectic groups*

In 2012, Gan and Savin established the local Langlands correspondence (LLC) for metaplectic groups. They transferred LLC for odd special orthogonal groups, via the theta correspondence. LLC should satisfy a lot of natural properties, and local intertwining relation (LIR) is one of them. We can also transfer LIR by applying the mixed model, which was introduced by Gan and Ichino, to our case.

10:45 – 11:45 山本 祐輝 (東京大学) / Yuki Yamamoto (University of Tokyo)

$GL_N$  の内部形式の超尖点表現の type について

*On the types for supercuspidal representations of inner forms of  $GL_N$*

Let  $G$  be the multiplicative group of a central simple algebra over a non-Archimedean local field. When we consider smooth representations of  $G$ , the theory of types is useful. A type is an irreducible smooth representation of some compact open subgroup in  $G$ , which can classify irreducible representations of  $G$  in a certain sense. I will explain types and discuss the existence and uniqueness of types. In particular, I will show that for an irreducible supercuspidal representation  $\pi$ ,  $[G, \pi]_G$ -types defined over some maximal compact subgroup in  $G$  are unique up to  $G$ -conjugation under some unramifiedness assumption on a simple stratum for  $\pi$ .

12:00 – 13:00 松村 英樹 (慶應義塾大学) / Hideki Matsumura (Keio University)

*Infinitely many hyperelliptic curves with exactly two rational points*

In arithmetic geometry, it is one of central problems to determine the sets of rational points on algebraic varieties. In this talk, we construct some families of infinitely many hyperelliptic curves with exactly two rational points. The key tools of the proof are the 2-descent argument, the Lutz-Nagell type theorem for hyperelliptic curves (Grant's theorem) and the Richelot isogeny. This is based on a joint work with Yoshinosuke Hirakawa (arXiv: 1904.00215v2).

14:30 – 15:30 巖 冬 (大阪大学) / Dong Yan (Osaka University)

剰余可約な肥田変形における岩澤不変量の変動について

*Variation of the two-variable Iwasawa invariant in residually reducible Hida deformation*

The two-variable Iwasawa theory for Hida deformation is studied by Ochiai, Skinner-Urban and Ochiai-Shimomoto. Especially, they proved the two-variable Iwasawa main conjecture partially when the residual representation is irreducible.

In this talk, I study the two-variable Iwasawa theory when the residual representation is reducible. One of subtle points in this case is that the Selmer group and its characteristic ideal depend on the choice of lattices of the two-variable Galois representation associated to Hida family. I show that there is a one-to-one correspondence between the set of all characteristic ideals and the set of all factors of the reflexive closure of Eisenstein ideal. Combining this result with a recent result of Bellaïche-Pollack, we give some examples where the two-variable Iwasawa main conjecture for residually reducible Hida deformation is true.

15:45 – 16:45 村上 和明 (慶應義塾女子高等学校) / Kazuaki Murakami (Keio Girls Senior High School)

*On the cyclicity of the unramified Iwasawa modules of the maximal multiple  $\mathbb{Z}_p$ -extensions over imaginary quadratic fields*

For an odd prime number  $p$ , we study the number of generators of the unramified Iwasawa modules of the maximal multiple  $\mathbb{Z}_p$ -extensions over Iwasawa algebras. In this talk, for an imaginary quadratic field, we give a necessary and sufficient condition for the Iwasawa module to be cyclic under several assumptions and give numerical examples. Moreover, as an application, we give an upper bound of  $\lambda$ -invariants for infinitely many  $\mathbb{Z}_p$ -extensions of an imaginary quadratic field. This is joint work with Takashi Miura, Rei Otsuki, and Keiji Okano.

17:00 – 18:00 片岡 武典 (慶應義塾大学) / Takenori Kataoka (Keio University)

楕円曲線の同変岩澤理論

*Equivariant Iwasawa theory for elliptic curves*

Iwasawa theory, which began with the study of ideal class groups, has now quite many extensions and refinements. In this talk, we discuss abelian equivariant Iwasawa theory for elliptic curves over  $\mathbb{Q}$ . Firstly, we formulate equivariant main conjectures in this situation. Secondly, under certain assumptions, we prove one divisibility via Euler system machinery. Finally, as an application, we prove a case of a conjecture of Mazur-Tate.

## 7月12日(金)

9:30 – 10:30 木村 昭太郎 (早稲田大学) / Shotaro Kimura (Waseda University)

*Modular differential equations for skew holomorphic Jacobi forms*

In 1998, Kaneko and Zagier introduced modular differential equations for elliptic modular forms of one variable. The modular form solutions of these differential equations is related to the supersingular polynomial and given in terms of hypergeometric polynomials. In 2016, Kiyuna constructed modular differential equations and found its solutions for holomorphic Jacobi forms of two variables. I constructed modular differential equations for skew holomorphic Jacobi forms which is not holomorphic of two variables. These equations have similarities with the case of elliptic modular forms and differences from the case of holomorphic Jacobi forms. In this talk, I will begin with reviewing known results and talk about my recent results.

10:45 – 11:45 境 優一 (九州大学) / Yuichi Sakai (Kyushu University)

*Modular linear differential equations in general form*

It is well known that modular linear differential equations (MLDEs) appear as tools in studies related to supersingular elliptic curves and classifications of characters of vertex operator algebras (VOAs). In some cases, MLDEs give a certain correspondence between modular forms and characters of VOAs. In this talk, we determine the properties of coefficients of MLDEs of any order by using the definition of MLDEs only. Furthermore, we give a general expression of MLDEs under the natural assumption for non-compact (quasi)modular forms. This is a part of a joint work with K. Nagatomo and D. Zagier.

12:00 – 13:00 中屋 智瑛 (九州大学) / Tomoaki Nakaya (Kyushu University)

*On the inductive structure of certain quasimodular forms of depth  $\leq 4$*

The extremal quasimodular forms of weight  $k$  and depth  $r = 1, 2$  on  $SL_2(\mathbb{Z})$  were studied by Kaneko and Koike. These forms are defined inductively by using differential operators, and satisfy certain modular linear differential equations (MLDEs). In this talk, we will construct certain quasimodular forms of weight  $k$  and depth  $\leq 4$  on  $SL_2(\mathbb{Z})$  and prove that these forms satisfy corresponding MLDEs of order  $r + 1$ .