Algebraic Number Theory and Related Topics 2015
Abstracts of talks

November 30 (Mon)

10:00 – 10:45 Minoru Hirose (Kyoto University)
Shintani zeta functions and Gross conjecture

The Gross conjecture is a refinement of Gross-Stark conjecture. The Gross conjecture consists of two parts: "vanishing order" part and "exact value" part. The second part predicts a mysterious relation between Stickelberger element and regulator. In 2014, Dasgupta and Spiess proved the first part of the conjecture under some conditions. In this talk, we construct a "enhanced" Stickelberger element. By using this, we give an alternative proof of the Dasgupta-Spiess's theorem and propose a refinement of the second part of the Gross conjecture.

10:55 – 11:40 Kazuto Ota (Keio University)
Kato's Euler system and the Mazur-Tate refined conjecture of BSD type

Mazur-Tate proposed a conjecture which compares the Mordell-Weil rank of an elliptic curve to the order of vanishing of Mazur-Tate elements, which are analogues of Stickelberger elements.
In this talk we explain the conjecture, and under some relatively mild assumptions, we give a sketch of its proof based on certain divisibility of Kato's Euler system.

11:50 – 12:40 Florian Sprung (Princeton University/IAS)
Integral Euler systems and Iwasawa theory

For an elliptic curve E over $\mathbb{Q}$ and a prime p of good reduction, the Iwasawa main conjecture is an equality between an analytically constructed ideal, from special values of the Hasse-Weil L-function, and a corresponding algebraic one, from subfields of the cyclotomic $\mathbb{Z}_p$-extension $\mathbb{Q}_\infty$ of $\mathbb{Q}$. When p has ordinary reduction, this conjecture has become a theorem via works of Kato and Skinner–Urban. In the supersingular case, one inclusion had been known from works of Kato, Kobayashi, and others. Recently, Wan has proved the reverse inclusion when $a_p = 0$. We sketch a generalization of Wan's result to the general supersingular case. An essential ingredient in the proof is an integral version of the Euler system constructed by Kings, Loeffler, and Zerbes, which we use in a non-standard way.

14:10 – 14:55 Jun Ueki (Kyushu University/University of Tokyo)
Theory of genera and Iwasawa invariants for 3-manifolds

We firstly establish an analogue for 3-manifolds of Furuta's genus formula for number fields, as an application of the idele theory by Niibo (and myself) for 3-manifolds. Then, using our genus formula, we prove an analogue for branched $\mathbb{Z}_p$-covers of Iwasawa's theorem given in 1973 on μ-invariants. Furthermore, we show an analogue of Kida's formula on λ-invariants for a cover of degree p-power of branched $\mathbb{Z}_p$-covers. Iwasawa gave a proof for Kida's formula using the cohomology of units in 1981. By the analogy between units and 2-cycles, we give a similar proof using the cohomology of 2-cycles.

15:10 – 15:55 Kotaro Sugahara (Kyushu University)
Central extensions and reciprocity law for arithmetic surfaces

In 2003, Osipov constructed dimension two central extensions using Parshin-Beilinson adeles and established reciprocity laws for algebraic surfaces. This theory is related to K_2 theory and intersection theory for algebraic surfaces. Since K_2 theory does not work for (infinite part of) arithmetic surfaces, we develop a new theory of central extensions based on Arakelov theory, then

we use our adelic cohomology theory to establish the reciprocity law for arithmetic surfaces. This is a joint work with Prof. Weng.

16:10 – 17:00 Takao Komatsu (Wuhan University)
Bernoulli-Carlitz numbers and Cauchy-Carlitz numbers

In 1935 Carlitz introduced Bernoulli-Carlitz numbers as analogues of Bernoulli numbers for the rational function field. In this talk, we introduce Cauchy-Carlitz numbers as analogues of Cauchy numbers. By using Stirling-Carlitz numbers, we give their arithmetical and combinatorial properties and relations with Bernoulli-Carlitz numbers for the rational function field. Several new identities are also obtained by using Hasse-Teichmüller derivatives. (This talk is based upon a joint work with Hajime Kaneko)

December 1 (Tue)

9:30 – 10:15 Keiji Adachi (Kyushu University)
Distance to cusps and stability

In the study of totally real fields, Siegel introduced a distance from a modular point to a cusp and hence constructed corresponding fundamental domains. This distance was generalized to work for all number fields in Prof. Weng's study on non-abelian zeta functions. Motivated by this, working on product of rigid analytic upper half planes, we first construct new distances between modular points and cusps. With the help of the correspondence between modular points and rank two bundles over curves defined over finite fields, we then obtain the following theorem.

Theorem: A rank two bundle on a curve over finite fields is Mumford semi-stable if and only if the distances of its associated modular point to all cusps are no less than one.

10:30 – 11:15 Megumi Takata (Kyushu University)
The infinite base change lifting associated to an APF extension of a mixed characteristic local field

In the Langlands conjecture, the base change lifting is the counterpart on the automorphic side of the restriction functor on the Galois side. For a finite cyclic extension of a number field or a mixed characteristic local field, the lifting was constructed by Langlands for GL(2) and by Arthur-Clozel for GL(n). In this talk, we give such a lifting for a totally ramified $\mathbb{Z}_p$-extension of a mixed characteristic local field. By Kazhdan's theory of close fields, we can interpret this as an operation which maps an automorphic representation of GL(n) over a local field of mixed characteristic to that of positive characteristic.

11:30 – 12:30 Yuichiro Hoshi (RIMS, Kyoto University)
Introduction to inter-universal Teichmüller theory I (Survey)

The purpose of this series of three talks is to give an introductory survey of the inter-universal Teichmüller theory established by Shinichi Mochizuki. In this talk, which is the first talk in the series, the speaker explains a "rough statement" of the main theorem of inter-universal Teichmüller theory. Moreover, we discuss why a Diophantine result can be derived from the main theorem.

14:05 – 14:50 Yoshinosuke Hirakawa (Keio University)
On the descent of certain modular Calabi-Yau varieties via the Cynk-Hulek construction

In their paper published in 2007, S. Cynk and K. Hulek introduced a construction of higher dimensional Calabi-Yau varieties from lower dimensional ones, and constructed some examples of modular Calabi-Yau varieties, which are Calabi-Yau varieties whose L-functions are described in terms of L-functions of modular forms. In this talk, we combine their idea with the Weil restriction functors and fixed point free involutions on the so called K3 surfaces (Calabi-Yau surfaces), and

construct new examples of modular Calabi-Yau varieties whose field of definition is the field of rational numbers. We also explain the relation between these examples and a certain Calabi-Yau realization problem of Hecke eigen newforms proposed by B. Mazur and D. van Straten.

15:05 – 15:55 Hironori Shiga (Chiba University)
One visualization of Shimura's complex multiplication theorem via hypergeometric modular functions (joint work with Atsuhira Nagano)

In the work " Construction of class fields and zeta functions of algebraic curves "(1967) by Goro Shimura, it is proved that there exists a modular function (that is called canonical model) that enables to obtain a certain class field of some kind of CM field. In our talk we show that for the case of the CM field embedded into the quaternion algebra coming from the compact arithmetic triangle group we can determine the canonical model as a hypergeometric modular function in an explicit way. Moreover we give several examples of Hilbert class fields of such kind of CM fields coming from the triangle group $\Delta(3,3,5)$. For our work, we use Shimura's reciprocity law and the existence of the canonical model together with the result by K. Takeuchi (1977). To construct explicit examples we use the modular function for genus 4 pentagonal curves discovered by K. Koike (2003).

16:10 – 17:00 Keisuke Arai (Tokyo Denki University)
Points on Shimura curves rational over imaginary quadratic fields in the non-split case

For a rational indefinite quaternion division algebra B, let M^B be the associated Shimura curve. We consider the behavior of the points on M^B rational over a number field k when the discriminant $d(B)$ of B grows. Jordan proved that if k is an imaginary quadratic field of class number $\neq 1$, if k splits B and if $d(B)$ is sufficiently large, then M^B has no k-rational points. In this talk, I discuss the case where k does not split B, and give an analogous result by imposing a certain congruent condition on $d(B)$. I also explain the reason why the non-split case is difficult.

December 2 (Wed)

9:00 – 9:45 Kento Yamamoto (Chuo University)
On generators of the Chow group of zero-cycles on diagonal cubic surfaces over 3-adic fields

In this talk, we give an explicit construction of generators of the degree-zero part of the Chow group of zero-cycles on some diagonal cubic surfaces over 3- adic fields. A key point is to check that such cycles have non-zero values under the Brauer-Manin pairing. We will show this by calculating Hilbert symbols.

10:00 – 10:45 Wataru Kai (University of Tokyo)
A moving lemma in the affine space with modulus

We show the contravariance of the additive higher Chow group, introduced by Bloch and Esnault in 2003, with respect to morphisms of smooth affine schemes. It is defined in terms of algebraic cycles satisfying the " modulus condition," which concerns their behavior at the boundary. Therefore we need a new moving lemma which can manage the "modulus condition."

11:00 – 11:45 Hiroyasu Miyazaki (University of Tokyo)
On moving algebraic cycles with modulus of bounded degree

Friedlander and Lawson developed the theory of moving algebraic cycles of bounded degree. The moving lemma of this type plays an important role in Voevodsky's $\mathbb{A}^1$-homotopy theory of motives. Recently, a non-homotopical generalization of Voevodsky's $\mathbb{A}^1$ -homotopy theory of motives is studied by Kahn-Saito-Yamazaki. The aim of this talk is to present a modification of

the moving lemma of Friedlander-Lawson which might be used in the (future) non-homotopical theory of motives.

12:00 – 13:00 Yuichiro Hoshi (RIMS, Kyoto University)
Introduction to inter-universal Teichmüller theory II (Survey)

The purpose of this series of three talks is to give an introductory survey of the inter-universal Teichmüller theory established by Shinichi Mochizuki. In this talk, which is the second talk in the series, the speaker introduces the notion of a "mono-theta environment" which plays an essential role in the nonarchimedean local portion of inter-universal Teichmüller theory, i.e., in the theory of étale theta functions. The speaker also explains a "multiradial representation of étale theta functions" obtained by applying the theory of mono-theta environments.

December 3 (Thu)

9:30 – 10:15 Arata Minamide (RIMS, Kyoto University)
Indecomposability of various profinite groups arising from hyperbolic curves

It is well-known that various profinite groups arising from hyperbolic curves (e.g., the geometric/arithmetic fundamental group of a hyperbolic curve over a number field) are center-free. In this talk, I will introduce the notion of "indecomposability", which is also a group-theoretic property of profinite groups, and discuss the indecomposability of various profinite groups arising from hyperbolic curves. Finally, from the point of view of combinatorial anabelian geometry, I will pose the question: Does the Grothendieck-Teichmüller group GT satisfy the indecomposability? I will give an affirmative answer to a pro-l version of this question by applying a certain anabelian result over finite fields.

10:30 – 11:15 Yu Yang (RIMS, Kyoto University)
On the existence, geometry and p-ranks of vertical fibers of coverings of curves

Let R be a complete DVR with algebraically closed residue field of characteristic $p > 0$, X a stable curve over R with smooth generic fiber, and $f : Y \longrightarrow X$ a morphism of stable curves over R such that the morphism of generic fibers induced by f is a Galois étale covering. A closed point x of X is called a vertical point if $\dim f^{-1}(x) = 1$. In this case, $f^{-1}(x)$ is called the vertical fiber associated to x. We study the existence, the geometry and the p-ranks of vertical fibers under certain assumptions. As an application, we generalize a theorem of M. Raynaud concerning ordinariness of coverings to the case of stable curves.

11:30 – 12:30 Yuichiro Hoshi (RIMS, Kyoto University)
Introduction to inter-universal Teichmüller theory III (Survey)

The purpose of this series of three talks is to give an introductory survey of the inter-universal Teichmüller theory established by Shinichi Mochizuki. In order to establish the main theorem of inter-universal Teichmüller theory, one has to suitably globalize local theories at various primes of a number field. In this talk, which is the third talk in the series, the speaker explains the notion of a "Hodge theater" which is closely related to such a globalization. Finally, the speaker gives a brief sketch of the proof of the main theorem.

14:00 – 14:45 Yasuhiro Wakabayashi (University of Tokyo)
Ordinariness and duality of dormant opers

A dormant oper is a certain principal homogeneous space on a pointed stable curve in positive characteristic equipped with an integrable logarithmic connection. The study of dormant opers and their moduli may be linked to various fields of mathematics, e.g., the p-adic Teichmüller theory developed by Shinichi Mochizuki, Gromov-Witten theory, combinatorics of rational polytopes

(and spin networks), etc. The key of the linkages between them is to consider the ordinariness of dormant opers, which may be thought of as a generalization of the usual ordinariness of curves in positive characteristic. In this talk, we would like to give a quick review of the definition of a dormant oper and to present some related results, including the ordinariness property of dormant opers on abelian coverings of the generic curve. Also, we would like to propose a certain duality between the moduli of dormant opers of rank $n(< p)$ and the moduli of dormant opers of rank $p - n$.

15:00 – 15:45 Yu Iijima (RIMS, Kyoto University)
Difference between l-adic Galois representations and pro-l outer Galois representations associated to hyperbolic curves

Let l be a prime number, k a field of characteristic zero, and V an algebraic variety over k. By considering a natural action of the absolute Galois group of k on the maximal pro-l (respectively, the abelianized maximal pro-l) quotient of the geometric fundamental group of V, we may obtain a pro-l outer Galois representation (respectively, an l-adic Galois representation) associated to V. In this talk, we explain the *nonexistence* of an isomorphism between the image of the pro-l outer Galois representation associated to V and the image of the l-adic Galois representation associated to V, in the case where V is a *hyperbolic curve* and k satisfies a mild assumption. Also, if time permits, then we explain a pro-Σ analog of the above result, where Σ is a nonempty set of prime numbers.

16:00 – 17:00 Hidekazu Furusho (Nagoya University)
Galois action on knots (Invited)

I will show that the absolute Galois group of the rational number field acts non-trivially on 'the space of profinite knots' in a non-trivial way.

December 4 (Fri)

9:30 – 10:15 Ippei Nagamachi (University of Tokyo)
On a good reduction criterion for proper polycurves with sections

Oda and Tamagawa showed that whether a hyperbolic curve over a discrete valuation field, whose residue character is $p \geq 0$, has good reduction can be determined by its pro-p' outer Galois representation. In this talk, I give a higher dimensional version of this criterion, i.e., a good reduction criterion for proper polycurves with sections (successive extensions of family of proper curves of genus ≥ 1 with section), under some condition about genera and p. In the proof, I will construct a homotopy exact sequence of fibration for certain Tannaka duals, which plays a crucial role.

10:30 – 11:20 Yuji Odaka (Kyoto University)
Canonical Kähler metrics and arithmetics - Generalising Faltings height

In a recent e-print (1508.07716), we introduced an extension of the Faltings height (1983) for general arithmetic varieties, which was originally defined for abelian varieties. We show that the extension inherits most of basic properties observed for the original Faltings height, especially for "K-stable" case, and depicts my broader conjectural picture involving e.g. the Minimal Model Program. Indeed this "K-stability/ canonical Kähler metrics" explains naturality of the original definition for abelian varieties case.

11:35 – 12:25 Chris Hall (University of Wyoming)
Specialization and big monodromy

Let k be a number field and U/k be a smooth geometrically-connected curve. We consider a "compatible system" of lisse $\mathbb{F}_\ell$-sheaves on U and compare the monodromy $G_{t,\ell}$ of special fibers with the geometric monodromy G_ℓ^{geom} of the generic fiber. We explain how to show that "most" special fibers have "big" monodromy, e.g., that $G_{t,\ell}$ contains the derived subgroup of G_ℓ^{geom}.

13:50 – 14:40 Haoyu Hu (University of Tokyo)
Semi-continuity of total dimension divisors for l-adic sheaves

Let $\mathcal{F}$ be a lisse ℓ-adic sheaf on an open dense subset U of a smooth variety X of characteristic $p \neq \ell$. Its non-logarithmic ramification along generic points of $X - U$ gives its total dimension divisor supported on $X - U$. In this talk, we will show that, in a smooth fibration, the total dimension divisor of the restriction of an ℓ-adic sheaf on fibers is lower semi-continuous. It is a generalization of the same property for relative curves obtained by G. Laumon in 1970's in the geometric case. This is a joint work with E. Yang.

14:50 – 15:35 Kazuki Tokimoto (University of Tokyo)
Affinoids in the Lubin-Tate perfectoid space and some cases of the local Langlands correspondence

The non-abelian Lubin-Tate theory states that the cohomology of the Lubin-Tate tower realizes the local Langlands correspondence and the local Jacquet-Langlands correspondence. Motivated by this theory, Boyarchenko-Weinstein and Imai-Tsushima recently constructed families of affinoids in a certain limit space of the tower (the Lubin-Tate perfectoid space) and showed that the cohomology of the reduction of (a formal model of) each affinoid realizes the two correspondences for particular kinds of representations. In this talk, we give a systematic construction of a family of affinoids generalizing that of Imai-Tsushima in the "essentially tame" case and discuss the cohomology of the reductions.

15:45 – 16:40 Yoichi Mieda (University of Tokyo)
Non-abelian Lubin-Tate theory and Kloosterman sheaves

Let F be a non-archimedean local field. By the local Langlands correspondence, irreducible supercuspidal representations of $\mathrm{GL}_n(F)$ are parameterized by irreducible n-dimensional representations of the Weil group of F. This correspondence is constructed by using the ℓ-adic étale cohomology of the universal deformation space of a 1-dimensional formal $\mathcal{O}_F$-module of height n (non-abelian Lubin-Tate theory). If F has equal characteristic, the parameter attached to a simple supercuspidal representation can also be constructed by using the Kloosterman sheaf. In this talk, I will compare these two geometric constructions. If time permits, I will also give a perspective on the case of general reductive groups.

Program Committee: Hiroki Sumida-Takahashi (Tokushima Univ.), Yasuo Ohno (Tohoku Univ.), Takahiro Tsushima (Chiba Univ.)