第 20 回広島仙台整数論集会アブストラクト

7 月 13 日 (火)

13:30 – 14:30 板東 克之 (東京大学) /Katsuyuki Bando (University of Tokyo)

Geometric Satake equivalence in mixed characteristic and Springer correspondence

The geometric Satake correspondence is an equivalence between the category of equivariant perverse sheaves on the affine Grassmannian and the category of representations of the Langlands dual group. It is known that there is a mixed characteristic version of the Geometric Satake correspondence. The Springer correspondence is a correspondence between the category of equivariant perverse sheaves on the nilpotent cone and the category of representation of the Weyl group. In this talk, we will explain some relation between these two correspondences, including the mixed characteristic case.

14:45 – 15:45 島田 了輔 (東京大学) /Ryosuke Shimada (University of Tokyo)

Geometric Structure of Affine Deligne-Lusztig Varieties for GL_3

The Langlands correspondence, which contains class field theory as a special case, is one of the most important topics in number theory. Shimura varieties have been used, with great success, towards applications in the realm of the Langlands program. In this context, geometric and homological properties of affine Deligne-Lusztig varieties have been used to examine Shimura varieties and the local Langlands correspondence. In this talk we study the geometric structure of affine Deligne-Lusztig varieties $X_\lambda(b)$ for GL_3 and b basic. We completely determine the irreducible components of the affine Deligne- Lusztig variety. In particular, we classify the cases where all of the irreducible components are classical Deligne-Lusztig varieties times finite-dimensional affine spaces. If this is the case, then the irreducible components are pairwise disjoint.

16:00 – 17:00 植木 潤 (東京電機大学) /Jun Ueki (Tokyo Denki University)

Non-trivial L-functions of the Whitehead link, triangle groups, and roots of unity

In Iwasawa theory and Hida–Mazur theory, torsion modules associated with group representations play important roles. In our setting, the non-trivial L-function of the universal deformation implies that the associated module is torsion. We previously determined for each $n \in \mathbb{Z}$ all residual representations of the twist knot $J(2, 2n)$ with $L \neq 0$, characterized them by exceptional surgeries, and further proved that $L \doteq k_n^2$ in $O[[x - \alpha]]$ for a certain $k_n \in \mathbb{Z}[x]$ related to $3n-1$-th roots of unity. In this talk, we recover previous results for all n at once from a result on the Whitehead link W_1 as well as point out a theoretical background for L being square. A key is a surjective homomorphism $\pi_1(S^3 - W_1) \twoheadrightarrow \Gamma(3, 3, \infty)$ to a triangle group with a $3 + \dfrac{1}{2}$-dimensional origin. This is a joint work with Léo Bénard (GAU, Göttingen), Ryoto Tange (Waseda U.), and Anh T. Tran (U. Texas at Dallas).

7 月 14 日 (水)

10:00 – 11:00 齋藤 光一郎 (名古屋大学) /Koichiro Saito (Nagoya University)

単価 Nielsen ポリログと超幾何関数

Single-valued Nielsen polylogarithms and hypergeometric functions

This talk will focus on a new aspect of Nielsen polylogarithms, which were introduced by Nielsen in 1909 in the context of generalizing classical polylogarithms. We will show that Nielsen polylogarithms represent the meta-abelian quotient of a fundamental solution of the one-variable KZ equation, and that the generating series of certain single-valued analogues of them can be expressed in terms of Gauss hypergeometric functions. We will also give an explicit closed formula for each single-valued Nielsen polylogarithm.

11:15 – 12:15 竹平 航平 (東北大学) /Kohei Takehira (Tohoku University)

力学系のゼータ関数の有理性と Woods Hole 固定点定理

Rationality of dynamical zeta function and Woods Hole fixed point formula

For a one variable rational function ϕ over a field K, we can define a discrete dynamical system by regarding ϕ as a self morphism over $\mathbb{P}^1$. There are many studies of the zeta function associated with a discrete dynamical system such as the Artin-Mazur zeta function and the Ruelle zeta function. In 1995, Hatjispyros and Vivaldi defined a dynamical zeta function for rational functions using multipliers of periodic points, that is, an invariant which indicates local behavior of dynamical systems. In this talk, we prove rationality of dynamical zeta functions of this type for a large class of rational functions. The proof uses Woods Hole fixed point formula and the trace of a linear map acting on cohomology of a coherent sheaf on $\mathbb{P}^1$.

13:30 – 14:30 冨田 拓希 (慶應義塾大学) /Takuki Tomita (Keio University)

絶対ゼータ関数の絶対 Euler 積表示について

The absolute Euler product representations of the absolute zeta functions

Let X be a scheme of finite type over $\mathbb{Z}$ satisfying a certain condition. The absolute zeta function for X is defined as the limit as $p \to 1$ of the congruent zeta function for X, which is the exponential of the generating function of the number of $\mathbb{F}_{p^m}$-rational points of X. In 2016, Kurokawa suggested that the absolute zeta function for a general scheme should have an infinite product structure (the absolute Euler product). In this talk, we will give a formulation of his suggestion using a torsion free Noetherian $\mathbb{F}_1$-scheme defined by Connes and Consani. Moreover, we will explain that each factor of the absolute Euler product is explicitly determined in terms of the function which interpolates the number of $\mathbb{F}_{p^m}$-rational points.

14:45 – 15:45 臺信 直人 (慶應義塾大学) /Naoto Dainobu (Keio University)

On ideal class groups of number fields associated to mod p modular Galois representations

Many people have noticed the existence of various relations between ideal class groups of number fields and Tate-Shafarevich groups of elliptic curves. For example, recently Prasad and Shekhar proved the following result. Let E be an elliptic curve over $\mathbb{Q}$ and $K = \mathbb{Q}(E[p])$ the p-th division field of E for an odd prime p. Under some assumptions, they showed that if the p-torsion part of the Tate-Shafarevich group of E has dimension at

least 2 over $\mathbb{F}_p$, then the ideal class group of K has $E[p]$ as its quotient $\mathrm{Gal}(K/\mathbb{Q})$-module. In this talk, we introduce a generalization of the above result for number fields attached to mod p modular Galois representations using setups on Bloch-Kato's Selmer groups and Tate-Shafarevich groups.

16:00 – 17:00 森 政興 (大阪大学) /Masaoki Mori (Osaka University)

$\mathbb{A}^1$ 代数トポロジーにおける Galois 理論

Galois theory for $\mathbb{A}^1$-algebraic topology

In Galois theory of the covering spaces, it is known that there exists canonical one-to-one correspondence between the set of isomorphic classes of coverings of a pointed topological space (X, x) and the set of conjugacy classes of subgroups of the fundamental group $\pi_1(X, x)$. In this talk, we give an analogue of this theory in $\mathbb{A}^1$-algebraic topology which was constructed by F. Morel and V. Voevodsky. First, we explain the notion of simplicial coverings of simplicial sets. For any fibrant connected pointed simplicial set (B, b), we present one-to-one correspondence between the set of subgroups of $\pi_1(B, b)$ and the weakly equivalent class of simplicial coverings of (B, b). Next, we talk about the generalization for simplicial sheaves of the above correspondence. Finally, for any fibrant $\mathbb{A}^1$-connected pointed simplicial sheaf $(\mathscr{X}, x)$, we give a bijective correspondence between the $\mathbb{A}^1$-weak equivalence classes of $\mathbb{A}^1$-coverings of $(\mathscr{X}, x)$ and subsheaves of $\mathbb{A}^1$-fundamental group of $(\mathscr{X}, x)$.

7 月 15 日 (木)

10:00 – 11:00 村上 友哉 (東北大学) /Yuya Murakami (Tohoku University)

モジュラー j 関数のサイクル積分の不良近似数への拡張

Extended-cycle integrals of the j-function for badly approximable numbers

Cycle integrals of the j-function are expected to play a role in the real quadratic analog of singular moduli. However, it is not clear how one can consider cycle integrals as a "continuous" function on real quadratic numbers. In this talk, we extend the definition of cycle integrals of the j-function from real quadratic numbers to badly approximable numbers to seek an appropriate continuity. We also give some explicit representations for extended-cycle integrals in some cases which can be considered as a partial result of continuity of cycle integrals.

11:15 – 12:15 沖 泰裕 (東京大学) /Yasuhiro Oki (University of Tokyo)

On the connected components of Shimura varieties for CM unitary groups in odd variables

Let (G, X) be a Shimura datum. Take a prime number p and a Bruhat–Tits subgroup K_p of $G(\mathbb{Q}_p)$. Consider the projective limit of the sets of connected components of Shimura varieties for (G, X) whose level at p are given by K_p. It is eqipped with the prime-to-p Hecke action. Then we discuss the question whether the above action is transitive, which is motivated by the theory of mod p reductions of Shimura varieties. In this talk, we give infinitely many projective systems of the Shimura varieties for CM unitary groups in odd variables for which the considering question is negative. To achieve this goal, we study a question related to the weak approximation on certain tori over $\mathbb{Q}$.

13:30 – 14:30 今井 湖都 (東京大学) /Koto Imai (University of Tokyo)

正標数の局所体上の冪零度最大のある有限次 Galois 拡大の分岐群

Ramification groups of some finite Galois extensions of maximal nilpotency class over local fields of positive characteristic

Galois extensions of local fields is one of the most important subjects in the field of number theory. A ramification filtration is a filtration of a Galois group used to investigate the ramification of the extension. It is particularly useful when the extension is wildly ramified. In this talk, we examine the ramification groups of finite Galois extensions over complete discrete valuation fields of characteristic $p > 0$. Brylinski calculated the ramification groups in the case where the Galois groups are abelian. We extend the results of Brylinski to some non-abelian cases where the Galois groups are of order $\leq p^{p+1}$ and of maximal nilpotency class.

14:45 – 15:45 金井 和貴 (新潟大学) /Kazuki Kanai (Niigata University)

Norm one tori and Hasse norm principle

Let k be a global field, K/k be a finite extension and $\mathbb{A}_K^\times$ be the idele group of K. We say that the Hasse norm principle (HNP) holds for K/k if $\mathrm{Obs}(K/k) := (N_{K/k}(\mathbb{A}_K^\times) \cap k^\times)/N_{K/k}(K^\times) = 1$ where $N_{K/k}$ is the norm map. In 1931, Hasse proved that if K/k is cyclic then the HNP holds but does not hold for bicyclic extension $\mathbb{Q}(\sqrt{-39}, \sqrt{-3})/\mathbb{Q}$. The HNP for Galois extensions was investigated by Tate, Gerth, Gurak, etc. However, for non-Galois extensions, very little is known about the HNP. In this talk, we give a method for computing $H^1(k, \mathrm{Pic}\,\overline{X})$ where X is a smooth k-compactification of the norm one torus $T = R^{(1)}_{K/k}(\mathbb{G}_m)$ by the computer algebra system GAP. Using this, we determine a necessary and sufficient condition for the HNP for K/k with $[K : k] \leq 15$ by combining the results of Ono and Voskresenskii in the 1960s, and of Drakokhrust and Platonov in the 1980s. As applications of the results, we get the group $T(k)/R$ of R-equivalence classes over a local field k via Colliot-Thélène and Sansuc's formula and the Tamagawa number $\tau(T)$ over a number field k via Ono's formula $\tau(T) = |H^1(k, \widehat{T})|/|Ш(T)|$. This is the joint work with Akinari Hoshi (Niigata University) and Aiichi Yamasaki (Kyoto University).

16:00 – 17:00 室谷 岳寛 (京都大学) /Takahiro Murotani (Kyoto University)

完備離散付値体の遠アーベル幾何学

A study on anabelian geometry of complete discrete valuation fields

Anabelian geometry has been developed over a much wider class of fields than Grothendieck, who is the originator of anabelian geometry, conjectured. More precisely, he considered that anabelian geometry should be developed over fields finitely generated over prime fields. However, Mochizuki gave some anabelian results over p-adic fields. On the other hand, algebraically closed fields of characteristic 0 are typical examples of fields which are not suitable for the base fields of anabelian geometry. So, it is natural to ask the following question: What kinds of fields are suitable for the base fields of anabelian geometry? In this talk, we consider this problem for higher local fields and mixed-characteristic complete discrete valuation fields with perfect residue fields.

7 月 16 日 (金)

9:30 – 10:30 李 公彦 (東京大学) /Kimihiko Li (University of Tokyo)

Prismatic and q-crystalline sites of higher level

Two new p-adic cohomology theories, called prismatic cohomology and q-crystalline cohomology, were defined for generalizing crystalline cohomology and they recover most known integral p-adic cohomology theories. On the other hand, higher level crystalline cohomology was defined for constructing p-adic cohomology theory over a ramified base. In this talk, for a positive integer m, we will give a construction of the level m primastic and q-crystalline sites and prove a certain equivalence between the category of crystals on the m-prismatic site or the m-q-crystalline site and that on the usual prismatic site or the usual q-crystalline site, which can be regarded as the prismatic analogue of the Frobenius descent. We will also prove the equivalence between the category of crystals on the m-prismatic site and that on the $(m-1)$-q-crystalline site.

10:45 – 11:45 石田 哲也 (佐賀大学) /Tetsuya Ishida (Saga University)

Local ε-conjecture for de Rham (φ, Γ)-modules and p-adic differential equations

(φ, Γ)-modules are defined by Fontaine as a generalization of p-adic representations of the absolute Galois group of a p-adic number field. Nakamura formulated a conjecture for (φ, Γ)-modules which generalizes Kato's local ε-conjecture for p-adic representations. On the other hand, for a (φ, Γ)-module satisfying the de Rham condition, Berger constructed a simpler p-adic differential equation. In this talk, we explain that the conjecture for the cyclotomic deformation of any de-Rham (φ, Γ)-module can be deduced from the one for the cyclotomic deformation of the corresponding p-adic differential equation. This is a joint work with Kentaro Nakamura.

12:00 – 13:00 町出 智也 (国立情報学研究所) /Tomoya Machide (National Institute of Informatics)

ブール多項式の連立方程式に関する公式について

On a formula for systems of Boolean polynomial equations

A Boolean polynomial is a congruence class of the polynomial ring over the finite field of characteristic 2, and identified with a Boolean function on the two-element set consisting of true and false. The Boolean polynomial is preferred in mathematics, and appears in algebraic geometry, Grobner basis, and cryptography. The Boolean function is preferred in computer science, and used to discuss the computational complexity exemplified by NP problem. In any areas, solving a system (or ideal) of Boolean polynomial equations is a main subject. In this talk, we prove a formula for systems of Boolean polynomial equations. The formula has both operations of conjunction (logical AND) and disjunction (logical OR) recursively, and conforms to De Morgan's duality. The formula comes from the distributivity of AND and OR. We also give its applications to the computational complexity using a parameter which is similar to the bandwidth of the matrix.