# 第 22 回広島仙台整数論集会アブストラクト

**7 月 11 日 (火)**

10:00 – 10:45 青木 謙典 (京都大学) / Kensuke Aoki (Kyoto University)

$A_{\mathrm{inf}}$-コホモロジーと半安定形式スキームの de Rham-Witt 複体との間の $p$-進 Cartier 同型 / A $p$-adic Cartier isomorphism between the $A_{\mathrm{inf}}$-cohomology and de Rham-Witt complexes for semistable formal schemes

In the $p$-adic Hodge theory, Bhatt-Morrow-Scholze constructed the $A_{\mathrm{inf}}$-cohomology theory for smooth formal schemes and show $p$-adic Cartier isomorphisms, which is applied to prove crystalline comparison theorem for the $A_{\mathrm{inf}}$-cohomology. On the other hand, Česnavičius-Koshikawa constructed the $A_{\mathrm{inf}}$-cohomology theory for semistable formal schemes over the ring of integers of $\mathbb{C}_p$. We prove the $p$-adic Cartier isomorphism between the $A_{\mathrm{inf}}$-cohomology and de Rham-Witt complexes for semistable formal schemes, extending the result of Bhatt-Morrow-Scholze in the smooth case.

11:00 – 11:45 浅山 拓哉 (東京工業大学) / Takuya Asayama (Tokyo Institute of Technology)

関数体の Kummer 忠実性 / Kummer-faithfulness for function fields

Kummer-faithuful fields, introduced by Mochizuki, form an important class in anabelian geometry. They are defined by the triviality of the divisible parts of the Mordell-Weil groups of semi-abelian varieties. In this talk, the speaker introduces the notion of Drinfeld-Kummer-faithful fields, which is a function field analogue of the notion of Kummer-faithful fields, using Drinfeld modules. A sufficient condition for a Galois extension of a function field to be Drinfeld-Kummer-faithful is provided in terms of ramification theory. Inspired by Ozeki-Taguchi's examples of highly Kummer-faithful fields, the speaker gives some examples of Drinfeld-Kummer-faithful fields.

12:00 – 12:45 後藤 新裕 (九州大学) / Akihiro Goto (Kyushu University)

The conditions of vanishing central value of Hasse-Weil $L$-function of a certain family of CM elliptic curve in Eisenstein integers case

The condition under which the central value of the Hasse-Weil $L$-function of an elliptic curve vanishes is very important and is also related to the BSD conjecture. We gave various characterizations for the infinite family $\{E_\lambda\}$ of CM elliptic curves on $\mathbb{Q}(\sqrt{-3})$ for which $L(1, E_\lambda) = 0$. It is described by rational numbers, which are analogues of Bernoulli numbers called Bernoulli-Hurwitz type numbers. We also obtained a congruence relation between the order of the Tate-Shafarevich group of $E_\lambda$ and the Bernoulli-Hurwitz type number under $L(1, E_\lambda) \neq 0$, which will be also presented as well. This talk is based on the joint work with Ryota Shii.

14:00 – 14:45 足立 大雅 (九州大学) / Taiga Adachi (Kyushu University)

2 次捻りによる保型形式の $L$ 関数の中心値の代数的部分の 2 進付値の変化について / The 2-adic valuations of the algebraic central $L$-values of quadratic twists of modular forms

Calculating the algebraic central $L$-values of quadratic twists of a Hecke eigenform is important in number theory. Recently, Shuai Zhai, Chao Li and Li Cai obtained the 2-adic valuation of the algebraic part under certain conditions by using modular symbols. We succeeded in calculating the 2-adic valuations for a new class of quadratic twists. This talk

is based on the joint work with Keiichiro Nomoto (Koden Electronics Co., Ltd.) and Ryota Shii (Kyushu university).

15:00 – 15:45  Abhinandan (University of Tokyo)

Prismatic $F$-crystals and Wach modules

For an unramified extension $F/\mathbb{Q}_p$ with perfect residue field, by works of Fontaine, Colmez, Wach and Berger it is known that the category of Wach modules over a certain integral period ring $\mathbb{A}_F^+$ is equivalent to the category of lattices inside crystalline representations of $G_F$, i.e. the absolute Galois group of $F$. Moreover, by recent work of Bhatt and Scholze, we also know that lattices inside crystalline representations of $G_F$ are equivalent to the category of prismatic $F$-crystals over $O_F$, the ring of integers of $F$. In this talk, we will present a direct construction of the categorical equivalence between Wach modules over $\mathbb{A}_F^+$ and prismatic $F$-crystals over $O_F$. If time permits, we will also mention generalization of our construction to the imperfect residue field case as well as to the relative case.

16:00 – 16:45  池田 香凜 (九州大学) / Karin Ikeda (Kyushu University)

On real zeros of the Hurwitz zeta function

In this talk, I will present a solution to the problem of real zeros of the Hurwitz zeta function, which has remained unsolved in previous studies. After reviewing the works of Spira, Nakamura, Matsuzaka, and Endo-Suzuki, I discuss the remaining case, namely the zeros in the interval $(-4, 0)$. I also present an observation of a curious behavior of a family of polynomials used in the proof.

17:00 – 17:45  井上 絢太郎 (京都大学) / Kentarou Inoue (Kyoto University)

Degeneration of $p$-divisible groups in terms of logarithmic geometry

The moduli space of abelian varieties are compactified by Faltings-Chai. Recently, Kajiwara-Kato-Nakayama realized this compactification as the moduli space of log abelian varieties which are degenerated objects in the world of logarithmic geometry. In the study of the mod $p$-fiber of this compactified moduli space, log $p$-divisible groups occur as degenerated objects of $p$-divisible groups. In this talk, we prove that log $p$-divisible groups over log regular bases admit slope filtrations up to isogeny.

## 7 月 12 日 (水)

10:00 – 10:45  石塚 康介 (東北大学) / Kosuke Ishizuka (Tohoku University)

The Goldstine theorem in the non-archimedean case

In functional analysis over $\mathbb{R}$ or $\mathbb{C}$ (which we call "classical analysis"), the Goldstine theorem says that the closed unit ball of a Banach space is dense in the closed unit ball of the bidual space for the weak star topology. On the other hand, Schikhof proved that the Goldstine theorem also holds in the non-archimedean case, if we replace the closed unit ball with the open unit ball. Then the question arises whether the exactly same statement as the classical Goldstine theorem holds in the non-archimean case. In this talk, we discuss the Goldstine theorem from the perspective of the theory of edged sets and that of compactoids which are analogous to compactness. As a result, we obtain a new result about the Goldstine theorem when the coefficient field is spherically complete.

11:00 – 11:45 石塚 伶 (東京工業大学) / Ryo Ishizuka (Tokyo Institute of Technology)

On the commutative ring-theoretic structure of the perfectoidization of semiperfectoid rings

The perfectoidization was introduced by Bhatt-Scholze as a mixed characteristic analogue of the perfection of rings of positive characteristic. In general, this is only a complex of some derived category, but for example the perfectoidization of semiperfectoid rings gives an honest ring. On the other hand, the notion of $p$-root closure was introduced by Roberts in commutative ring theory. This is a simple closure operation that existed before perfectoid rings appeared and is closely related to the structure of the Fontaine ring (now often called tilting). In this talk, we show that the perfectoidization of the semiperfectoid ring $S$ is (almost) isomorphic to the $p$-adic completion of the $p$-root closure of $S$.

12:00 – 12:45 石井 竣 (慶應義塾大学) / Shun Ishii (Keio University)

On pro-$p$ outer Galois representations associated to once-punctured CM elliptic curves

Let $p$ be an odd prime. Sharifi proved that, under the Deligne-Ihara conjecture (now a theorem of Brown), the kernel of the pro-$p$ outer Galois representation associated to the thrice-punctured projective line corresponds to the maximal pro-$p$ extension of $\mathbb{Q}(\mu_p)$ unramified outside $p$ if $p$ is regular. In this talk, we discuss an analogue of his result for imaginary quadratic fields of class number one by considering once-punctured CM elliptic curves instead of the thrice-punctured projective line.

14:00 – 14:45 川村 花道 (東京理科大学) / Hanamichi Kawamura (Tokyo University of Science)
1/2 多重ゼータ値の重み付き和公式とその類似 / Weighted sum formula for half multiple zeta values and its variants

The weighted sum formula for multiple zeta values has been found in various forms as a weighted analogue of Granville's sum formula. In this talk, we consider a new type of formulas weighted by powers of 2 that have been discovered for the special cases of Yamamoto's interpolated multiple zeta values using some formulas of the generalized hypergeometric series. This is a joint work with Takumi Maesaka (Kanazawa Institute of Technology) and Masataka Ono (Waseda University).

15:00 – 15:45 角野 裕太 (東北大学) / Yuta Kadono (Tohoku University)
川島関数の積分表示と Taylor 係数 / An integral representation of Kawashima function and its Taylor coefficients

It is known that there are many $\mathbb{Q}$-linear relations among multiple zeta values. In particular, the Kawashima relation is one of families of relations that is expected to generate all $\mathbb{Q}$-linear relations among multiple zeta values and can be described using the Taylor coefficients of the Kawashima function. In this talk, we generalize the multiple integrals, which have been studied for a long time in transcendental number theory, to give a multiple integral representation of the Kawashima function. Furthermore, we express the Taylor coefficients of the Kawashima function in terms of multiple zeta values with explicit indices.

16:00 – 16:45 隈部 哲 (九州大学) / Satoshi Kumabe (Kyushu University)

Supercongruences for the Appell functions and elliptic curves with complex multiplication

There are congruence relations modulo a power of a prime number $p$ arising from the coefficients of a power series expansion of an invariant differential on an elliptic curve over

$\mathbb{Q}$. These congruence relations are called Atkin and Swinnerton-Dyer congruences. Furthermore, these congruence relations hold modulo a higher power of $p$ in a particular case. Then such congruence relations are called supercongruences. In this talk, I will introduce supercongruences for certain hypergeometric functions of two variables and elliptic curves with complex multiplication.

17:00 – 17:45 松月 大知 (名古屋大学) / Daichi Matsuzuki (Nagoya University)

On non-vanishing of higher genus multiple zeta values

Thakur introduced multiple zeta values associated with the rational function field over a finite field and proved that they are non-zero by using a result of Sheats. In the talk, we prove that multiple zeta values associated with the function field of a higher genus curve, which were introduced by Lara Rodriguez and Thakur, are non-zero under a certain condition on the Weierstrass gap sequence at a fixed rational point $\infty$. By arguments similar to those performed by Thakur in the proof for the case of the rational function field, we explicitly calculate the $\infty$-adic valuation of a given multiple zeta value and we obtain the result by the calculation.

## 7 月 13 日 (木)

09:00 – 09:45 小林 雅人 (神奈川大学) / Masato Kobayashi (Kanagawa University)

Applications of Ramanujan's work on Eisenstein series

Ramanujan (1916) expressed quotients of certain $q$-series as polynomials of the Eisenstein series $P, Q, R$ and derived what we call Ramanujan's differential equations. We continue this research with the variants of Eisenstein-type series which Hahn (2007) recently introduced. We also prove new formulas of convolution sums for divisor sum functions as subsequent work of Cheng-Williams (2004) and Huard-Ou-Spearman-Williams (2002).

10:00 – 10:45 松本 圭峰 (大阪大学) / Keiho Matsumoto (Osaka University)

非可換 $p$-進 Hodge 理論について / On non-commutative $p$-adic Hodge theory

In this talk, we will discuss the crystalline conjecture for non-commutative algebraic varieties, and explain our results about the non-commutative version of Bhatt-Morrow-Scholze's Breuil-Kisin cohomology theory. Time permitting, as an application to arithmetic geometry, we will talk about the relation between our results and the crystalline comparison theorem for open varieties proved by Yamashita.

11:00 – 11:45 松本 晃二郎 (東京大学) / Kojiro Matsumoto (University of Tokyo)

On the potential automorphy and the local-global compatibility for the monodromy operators at $p \neq l$ over CM fields

Let $F$ be a CM field and $n$ be a positive integer. About the global Langlands correspondence for $\mathrm{GL}_{n,F}$, there are many general results in regular essentially conjugate self-dual cases. For example, we have the complete local-global compatibility and the potential automorphy for many general Galois representations. On the other hand, for any regular C-algebraic cuspidal automorphic representation $\pi$ of $\mathrm{GL}_n(\mathbb{A}_F)$, Harris-Lan-Taylor-Thorne constructed the Galois representation corresponding to $\pi$, Ila Varma proved the local-global compatibility up to semisimplification at $p \neq l$ and many people proved some automorphy lifting theorems and potential automorphy theorems for some special Galois representations. But we haven't proved the local-global compatibility for the monodromy operators

at $p \neq l$ and the potential automorphy for general Galois representations, unlike regular essentially conjugate self-dual cases. In this talk, we observe some relations between the potential automorphy of $l$-adic Galois representations of $F$ and the local-global compatibility for the monodromy operators at $p \neq l$. As applications, we prove the local-global compatibility for the monodromy operators at $p \neq l$ in many cases and the Ramanujan conjecture for regular C-algebraic cuspidal automorphic representations of $\mathrm{GL}_2(\mathbb{A}_F)$ by using some potential automorphy theorems.

12:00 – 12:45 村上 友哉 (九州大学) / Yuya Murakami (Kyushu University)

3 次元多様体の量子不変量, 偽テータ関数, $L$ 関数の特殊値 / Quantum invariants of 3-manifolds, false theta functions and special values of $L$-functions

Quantum invariants of 3-manifolds are topological objects defined from a mathematical physics viepoint. They also have deep connections with number theory. For instance, Zagier and others had been pointed out relation between the asymptotic expansions of quantum invariants and modular forms. Under this perspective, mathematical physicists Gukov-Pei-Putrov-Vafa conjectured a certain relationship between quantum invariants. In this talk, I will discuss a proof of this conjecture. The methods of proof yields relation and vanishing property of special values of certain $L$-functions.

14:00 – 14:45 室谷 岳寛 (東京工業大学) / Takahiro Murotani (Tokyo Institute of Technology)

有限体の無限次代数拡大体上の遠アーベル幾何学 / Anabelian geometry over infinite algebraic extensions of finite fields

The Grothendieck conjecture for hyperbolic curves over finite fields was solved affirmatively by Tamagawa and Mochizuki. Moreover, Tamagawa also proved (a "weak version" of) the Grothendieck conjecture for hyperbolic curves of genus zero over algebraic closures of finite fields. So, it is natural to consider anabelian geometry over (infinite) algebraic extensions of finite fields. In this talk, we consider to what extent we can develop anabelian geometry over these fields, and give some results (e.g., a generalization of Tamagawa's result).

15:00 – 15:45 中川 彬雄 (千葉大学) / Akio Nakagawa (Chiba University)

Multi-variable hypergeometric functions and algebraic varieties over finite fields

Hypergeometric functions over $\mathbb{C}$ satisfy many formulas and have integral representations. For one-variable hypergeometric functions over finite fields, finite field analogues of the integral representations are known. Through the integral representations and their analogues, hypergeometric functions over $\mathbb{C}$ and over finite fields are related with algebraic varieties. In this talk, we prove finite field analogues of some formulas and integral representations for Appell-Lauricella multi-variable hypergeometric functions. Furthermore, we express the numbers of rational points on some algebraic varieties in terms of Appell-Lauricella functions over finite fields. The speaker will also talk about the recent joint work with Ryojun Ito, Satoshi Kumabe and Yusuke Nemoto on Kampé de Fériet hypergeometric functions over finite fields.

16:00 – 16:45 沖 泰裕 (北海道大学) / Yasuhiro Oki (Hokkaido University)

On Tamagawa numbers of tori attached to CM algebras

The notion of Tamagawa numbers is one of the most important subject in arithmetic of

tori over number fields. In 1963, Takashi Ono predicted that all positive rational numbers are realized as the Tamagawa numbers of tori over number fields. This conjecture was resolved by Shin-ichi Katayama in 1985. In this talk, we define "tori attached to CM algebras", which are defined over the field of rational numbers, and give an analogue of Ono's conjecture/Katayama's theorem for such tori. More precisely, we explain that every power of 2 can be written as the Tamagawa number of a torus attached to a CM algebra. This talk is based on the joint work with Pei-Xin Liang, Hsin-Yi Yang and Chia-Fu Yu.

17:00 – 17:45 大塚 瑛介 (東北大学) / Eisuke Otsuka (Tohoku University)

Iterated integrals over some algebraic curves and their motivic interpretations

Multiple zeta values (MZVs for short) can be represented as iterated integrals of $\mathbb{Q}$-rational differential forms on $\mathbb{P}^1(\mathbb{C}) \setminus \{0, 1, \infty\}$. This representation allows us to consider MZVs geometrically, and this is one of the motivations for Deligne, Goncharov, Terasoma et al. to give a motivic interpretation of MZVs by using the fundamental group and mixed Tate motive theory. In this talk, we consider the values given by iterated integrals on other concrete algebraic curves over $\mathbb{Q}$ and explain some results. In particular, we give motivic interpretations of them by investigating relations with MZVs with levels. At this point, it is important to consider the base expansion and the Galois invariant part of the space of motivic MZVs. I will also explain it.

## 7 月 14 日 (金)

09:00 – 09:45 齋藤 陽平 (慶應義塾大学) / Yohei Saito (Keio University)

Theta lifts to certain cohomological representations of indefinite orthogonal groups

Howe and Tan investigated a degenerate principal series representation of indefinite orthogonal groups and completely described its composition series. Their works imply that there exists a unique unitalizable irreducible submodule, which is isomorphic to a non-tempered derived functor module. We study the theta liftings to this irreducible representation. In this talk, we introduce a new observation of the liftings of holomorphic automorphic forms on $\mathrm{Mp}_2(\mathbb{R})$ and, by combining it with the Borcherds' method, explicitly compute the Fourier expansion of resulting automorphic forms. As an application, we prove the square integrability of the liftings.

10:00 – 10:45 椎井 亮太 (九州大学) / Ryota Shii (Kyushu University)

On non-trivial $\Lambda$-submodules with finite index of the plus/minus Selmer group over anti-cyclotomic $\mathbb{Z}_p$-extension at inert primes

In Iwasawa theory, it is a basic problem that a Selmer group of elliptic curves has no non-trivial $\Lambda$-submodules with finite index. Regarding this problem, there is some research by R. Greenberg, B. D. Kim, and T. Kitajima–R. Otsuki. Recently, we solved this problem under some assumptions for the anticyclotomic $\mathbb{Z}_p$-extension of an imaginary quadratic field in which $p$ is inert. In this talk, we present the background of this research and prove this result.

11:00 – 11:45 島田 了輔 (東京大学) / Ryosuke Shimada (University of Tokyo)

Crystal bases and affine Deligne-Lusztig varieties

The notion of affine Deligne-Lusztig variety was first introduced by Rapoport, which plays an important role in understanding geometric and arithmetic properties of Shimura varieties. The geometric properties of affine Deligne-Lusztig varieties have been studied by many people, and one of the most interesting results is an explicit description of the set of irreducible components. Surprisingly, irreducible components of affine Deligne-Lusztig varieties are parametrized by a certain weight space of crystal bases. In this talk, I will talk about my recent work on the explicit construction of irreducible components from crystal bases.

12:00 – 12:45 高谷 悠太 (東京大学) / Yuta Takaya (University of Tokyo)

Equidimensionality of affine Deligne-Lusztig varieties in mixed characteristic

Affine Deligne-Lusztig varieties are a generalization of the special fiber of local Shimura varieties, such as Rapoport-Zink spaces. It is expected to have geometric properties similar to those of Newton strata of Shimura varieties. In this talk, we prove one of its expected properties, equidimensionality. In the equicharacteristic case, equidimensionality was proven by Hartl and Viehmann. However, affine Deligne-Lusztig varieties are perfect schemes in mixed characteristic, which makes it harder to study their geometric properties. We will explain obstacles to applying Hartl and Viehamann's arguments in mixed characteristic and present a way of resolving those issues.

14:00 – 14:45 山口 永悟 (東京工業大学) / Naganori Yamaguchi (Tokyo Institute of Technology)

遠アーベル幾何学の $m$ 次可解化を使った新たな展開について / New developments in anabelian geometry by using $m$-step reconstruction

In anabelian geometry, there is an important conjecture known as the Grothendieck conjecture. This conjecture states that the geometric properties of hyperbolic (algebraic) curves can be determined group-theoretically by studying their arithmetic fundamental groups. H. Nakamura, A. Tamagawa, S. Mochizuki, and J. Stix successfully proved this conjecture for fields of any characteristic (until 2009). Nevertheless, several open problems remain related to this conjecture. This talk focuses on one such problem, called the $m$-step solvable Grothendieck conjecture, which serves as the central theme. The $m$-step solvable Grothendieck conjecture concerns the group-theoretical reconstruction of geometric properties of hyperbolic (algebraic) curves by "near" abelian quotients of their fundamental groups – more precisely, by the maximal geometrically $m(\geq 2)$-step solvable quotient of their arithmetic fundamental groups. In this talk, we will present the $m$-step solvable Grothendieck conjecture and a part of its proof as obtained by the speaker.

15:00 – 15:45 山本 寛史 (東京大学) / Hirofumi Yamamoto (University of Tokyo)

$p$-通常的半整数重さ次数 2 ジーゲルモジュラー形式の空間の次元について / On the dimension of spaces of $p$-ordinary half-integral weight Siegel modular forms of degree 2

Hecke eigenforms whose eigenvalues at $p$ are $p$-adic units are called $p$-ordinary. The dimension of the space spanned by $p$-ordinary eigenforms over many reductive groups, including Siegel modular forms, is known to be bounded regardless of levels and weights. Furthermore, Hida showed that the dimension is bounded in the case of half-integral weight modular forms as well, using Shimura correspondence. We will show the similar result for half-integral weight Siegel modular forms of degree 2, using an analogy of Shimura correspondence for Siegel modular forms (Ibukiyama conjecture).