

第 19 回仙台広島整数論集会アブストラクト

2020 年 9 月 8 日 (火)~9 月 11 日 (金)

Zoom によるオンライン開催

9 月 8 日 (火)

10:00 – 11:00 松坂 俊輝 (名古屋大学) / Toshiaki Matsusaka (Nagoya University)

Rademacher 記号の双曲類似

Hyperbolic analogue of the Rademacher symbol

One of the most famous results of Dedekind is the transformation law of $\log \eta(z)$. In 1956, Rademacher introduced an $SL_2(\mathbb{Z})$ -conjugacy class invariant (integer-valued) function $\Psi(\gamma)$ by modifying Dedekind's result. We call it the *Rademacher symbol*. The symbol appears in several fields of mathematics. Inspired by Ghys' work on modular knots, Duke-Imamoğlu-Tóth (2017) constructed a hyperbolic analogue of the symbol $\Psi(\gamma)$.

In this talk, we give a new expression of their hyperbolic Rademacher symbol in terms of the geodesic integral of the hyperbolic Eisenstein series, and show some properties.

11:15 – 12:15 植木 潤 (東京電機大学) / Jun Ueki (Tokyo Denki University)

ツイスト結び目の非自明な L 不変量と 1 の $3n - 1$ 乗根

Non-trivial L -invariants of twist knots and $(3n - 1)$ -th roots of unity

We study irreducible SL_2 representations of the fundamental group of the exterior of the twist knot $J(2, 2n)$ with $n \in \mathbb{Z}$. Our main tools are the character variety, the (acyclic) Reidemeister torsion τ , and the Chebyshev polynomials. For the universal deformation over a CDVR O of a residual representation, the (algebraic) L -invariant is defined as a certain torsion invariant. Our results are as follows.

Over any field, every non-acyclic representation corresponds to $3n - 1$ -th root of unity. Non-acyclic representations are topologically characterized (factoring through a certain exceptional representation $+\alpha$). Every non-acyclic residual representation admits a universal deformation over $SL_2O[[x - \alpha]]$. The L -invariant is nontrivial iff the residual representation is non-acyclic. For a certain polynomial $k_n(x) \in \mathbb{Z}[x]$ relating to $3n - 1$ -th roots of unity, we have $L = (k_n)^2$ up to multiplication by units.

These results would shed new light on the analogy between Hida–Mazur's Galois deformation theory and Thurston's hyperbolic deformation theory. This talk is based on the joint work(s) with Ryoto Tange (Kogakuin U.), Anh T. Tran (U.Texas at Dallas), and Leo Benard (GAU, Göttingen).

13:30 – 14:30 前田 洋太 (京都大学) / Yota Maeda (Kyoto University)

ユニタリ型志村多様体上の特異点と小平次元への応用

On the singularities of unitary Shimura varieties and their applications to the Kodaira dimension

The Kodaira dimension of Shimura varieties has been studied by many people. Kondo showed that certain orthogonal Shimura varieties related with the moduli spaces of K3 surfaces have non-negative Kodaira dimension by using the Borchers lift. Recently, Gritsenko-Hulek-Sankaran proved there exist toroidal compactifications of orthogonal Shimura varieties which have canonical singularities and no branch divisors in the boundary components. As an application, they obtained better results on the Kodaira dimension than Kondo's results. In this talk, we consider similar problems for unitary Shimura varieties. We construct toroidal compactifications of some unitary Shimura varieties satisfying the above properties. Finally, we give an application to the Kodaira dimension of unitary Shimura varieties in terms of reflective modular forms.

14:45 – 15:45 高松 哲平 (東京大学) / Teppei Takamatsu (University of Tokyo)

On the finiteness of twisted forms of hyperkahler varieties

For a finite field extension L/K and a variety X over K , let $\text{Tw}_{L/K}$ be a set of isomorphism classes of varieties Y over K which are isomorphic to X after a base change to L . Such a variety Y is called a twisted form of X , and the diversity of twisted forms is one of the difficulties of the study of varieties over number fields. In this talk, we consider the finiteness of $\text{Tw}_{L/K}$ for K3 surfaces of characteristic away from 2, and hyperkähler varieties of characteristic 0. This work is a generalization of Cattaneo–Fu's work on real forms of hyperkähler varieties. We also give an application to the finiteness of derived equivalent hyperkähler varieties.

16:00 – 17:00 山田 一紀 (慶應義塾大学) / Kazuki Yamada (Keio University)

Hyodo–Kato theory with syntomic coefficients

Hyodo–Kato theory in the original sense is a theory of comparison between two p -adic cohomologies, the de Rham cohomology and the log crystalline cohomology. In this talk, we study coefficients in Hyodo–Kato theory under the rigid analytic reinterpretation given by our previous work with Veronika Ertl. We first introduce a notion of log overconvergent F -isocrystals as coefficients of the Hyodo–Kato cohomology, and give a rigidity theorem for them. Then we define a certain notion of coefficients of the p -adic Hodge cohomology, and study their fundamental properties, e.g. the independence from a choice of uniformizer, the relation with filtered (φ, N) -modules, and the admissibility.

9月9日(水)

10:00 – 11:00 清水 陵嗣 (京都大学) / Ryoji Shimizu (Kyoto University)

制限分岐版ノイキルヒ・内田の定理について

The Neukirch-Uchida theorem with restricted ramification

The Neukirch-Uchida theorem is one of the most important results in anabelian geometry. In this talk, I will discuss the following generalization of the Neukirch-Uchida theorem under some assumptions: “For $i = 1, 2$, let K_i be a number field and S_i a set of primes of K_i . We write $K_{i,S_i}/K_i$ for the maximal extension of K_i unramified outside S_i and G_{K_i,S_i} for its Galois group. If G_{K_1,S_1} and G_{K_2,S_2} are isomorphic, then K_1 and K_2 are isomorphic.” The assumptions include: the Dirichlet density of S_i is not zero for $i = 1, 2$; K_i is Galois over \mathbb{Q} for $i = 1, 2$; K_1 is totally imaginary; and so on.

11:15 – 12:15 山口 永悟 (京都大学) / Naganori Yamaguchi (Kyoto University)

The m -step solvable Grothendieck conjecture for genus 0 curves over finitely generated fields

In anabelian geometry, the Grothendieck conjecture is one of the most important problem. For this conjecture, many previous results were obtained so far (Tamagawa, Mochizuki, et al). As a variant, we can also consider the m -step solvable Grothendieck conjecture. Briefly, this is the problem of reconstructing the isomorphism class of a curve from the maximal geometrically m -step solvable quotient of the arithmetic fundamental group of the curve. For this variant, previous results were obtained by Nakamura and Mochizuki. In this talk, I will outline the proof of my result on the m -step solvable Grothendieck conjecture for curves of genus 0 over fields finitely generated over the prime field.

13:30 – 14:30 田中 秀宜 (東洋大学) / Hidenori Tanaka (Toyo University)

On the Euler product expressions of the absolute tensor products of the Dirichlet L -functions

In 1992 Kurokawa defined the absolute tensor product $(Z_1 \otimes_{\mathbb{F}_1} \cdots \otimes_{\mathbb{F}_1} Z_r)(s)$ of some zeta functions $Z_j(s)$ ($j = 1, \dots, r$) as a function which had zeros or poles only at $s = \rho_1 + \cdots + \rho_r$ where $\rho_j \in \mathbb{C}$ with $Z_j(\rho_j) = 0$ or ∞ and predicted it to have the expression by the Euler product over r -tuples of primes if $Z_j(s)$ was represented by the Euler product over primes for each $j \in \{1, \dots, r\}$. The validity of Kurokawa’s prediction has been confirmed in some cases, for example, the cases of the Hasse zeta functions of finite fields by Koyama and Kurokawa for $r = 2$, by Akatsuka for $r = 3$ and by Kurokawa and Wakayama for general r . Also, the case of the Riemann zeta function for $r = 2$ was first proved by Koyama and Kurokawa, and then by Akatsuka in a different way.

In this talk, I will explain the method to construct the Euler product expression of the absolute tensor product on the Dirichlet L -functions $L(s, \chi_j)$ ($j = 1, \dots, r$) which is a generalization of Akatsuka’s method for the Riemann zeta function and show that Kurokawa’s prediction is valid in the case of the Dirichlet L -functions for $r = 2$.

14:45 – 15:45 関川 隆太郎 (東京理科大学) / Ryutaro Sekigawa (Tokyo University of Science)
Rikuna's generic cyclic polynomial and the monogenity

Let K/k be an extension of number fields and $\mathcal{O}_K, \mathcal{O}_k$ be the rings of integers of K, k , respectively. It is said that an element α of K forms a power integral basis for K/k if $\mathcal{O}_K = \mathcal{O}_k[\alpha]$. In such a case, we say K/k is monogenic. For an odd prime l , we put $k = \mathbb{Q}(\omega)$ with $\omega = \zeta + \zeta^{-1}$ where ζ is a primitive l -th root of unity. We consider a cyclic extension K_s/k of degree l defined by Rikuna's generic cyclic polynomial, parameterized by $s \in \mathcal{O}_k$. In this talk, We introduce a sufficient condition for the monogenity of K_s/k and a proof that there exist infinitely many monogenic cyclic extensions K_s/k , although monogenic cyclic extensions of degree $l \geq 5$ over \mathbb{Q} are rather rare.

16:00 – 17:00 山田 智宏 (大阪大学) / Tomohiro Yamada (Osaka University)
On the problem of de Koninck

Let $\sigma(n)$ and $\gamma(n)$ denote the sum of divisors and the product of distinct prime divisors of n , called the radical or the squarefree kernel of n , respectively. De Koninck asked if the only solutions of $\sigma(n) = (\gamma(n))^2$ are $n = 1$ and 1782. We shall show that if $n \neq 1, 1782$ and $\sigma(n) = (\gamma(n))^2$, then there exist odd (not necessarily distinct) primes p, p' and (not necessarily odd) primes $q_i (1 \leq i \leq k)$ with $k \leq 3$ such that $p, p' \parallel n, q_i^2 \parallel n (1 \leq i \leq k)$ and $q_1 \mid \sigma(p^2), q_{i+1} \mid \sigma(q_i^2) (1 \leq i \leq k-1), p' \mid \sigma(q_k^2)$.

9月10日(木)

10:00 – 11:00 伊藤/Sprung Florian (アリゾナ州立大学) / Ito/Sprung Florian (Arizona State University)

楕円曲線の虚二次体の \mathbb{Z}_p^2 拡大における有理点の Mordell-Weil 階数

The Mordell-Weil rank of elliptic curves in \mathbb{Z}_p^2 -extensions of imaginary quadratic fields

The Mordell-Weil rank remains bounded in cyclotomic \mathbb{Z}_p -extensions of a number field K , even though there are infinite layers (and therefore the rank has infinite chances to increase). For $K = \mathbb{Q}$, this is the end of the story because the cyclotomic \mathbb{Z}_p -extension is the only one. When looking at the next simplest case – where K is a quadratic imaginary field – there are infinitely many \mathbb{Z}_p -extensions, and it is known that the rank remains bounded in most of them, while for one special extension (the anticyclotomic one), the rank may be unbounded. In joint work with A. Lei, we give an estimate for the growth of the rank in all the \mathbb{Z}_p -extensions by considering their compositum, the \mathbb{Z}_p^2 -extension of K .

11:15 – 12:15 野本 慶一郎 (九州大学) / Keiichiro Nomoto (Kyushu University)

On the rank of the elliptic curve $y^2 = x^3 + px$ and a recurrence formula

Let p be a prime number and E_p denote the elliptic curve $y^2 = x^3 + px$. It is known that for p which is congruent to 1, 9 modulo 16, the rank of E_p over \mathbb{Q} is equal to 0, 2. Under the condition that the Birch and Swinnerton-Dyer conjecture is true, we give a necessary and sufficient condition that the rank is 2 in terms of the constant term of a polynomial that is defined by a recurrence formula.

13:30 – 14:30 金村 佳範 (慶應義塾大学) / Yoshinori Kanamura (Keio University)

How to calculate the proportion of everywhere locally soluble diagonal hypersurfaces

Rational points on diagonal hypersurfaces have been studied for a long time in connection with the Waring problem and the Fermat's Last Theorem. In this talk, we give a strategy to calculate the proportion of everywhere locally soluble diagonal hypersurfaces. Here, we consider the family of diagonal hypersurfaces of the projective space for every fixed dimension and degree. Our strategy is based on the product formula established by Bright, Browning and Loughran. As working examples, we give the proportions of diagonal hypersurfaces with rational points for each dimension under a certain hypothesis on Brauer-Manin obstruction when they are quadratic and cubic. This is a joint work with Yoshinosuke Hirakawa at Keio University (arXiv:2003.11426).

14:45 – 15:45 中川 彬雄 (千葉大学) / Akio Nakagawa (Chiba University)

対角超曲面の Artin L 関数と有限体上の一般超幾何関数

Artin L -functions of diagonal hypersurfaces and generalized hypergeometric functions over finite fields

In general, if a finite group G acts on a variety V then we can define the Artin L -function of V associated to a character of G , and the congruence zeta function of V is decomposed to the Artin L -functions of V .

In this talk, we compute the Artin L -function of a diagonal hypersurface D_λ over a finite field associated to a character of a finite group acting on D_λ , and under some condition, express it in terms of hypergeometric functions and Jacobi sums over the finite field. As an application, we consider the Dwork hypersurfaces and obtain relations between certain hypergeometric functions over different finite fields.

16:00 – 17:00 隈部 哲 (九州大学) / Satoshi Kumabe (Kyushu University)

Dwork hypersurfaces of degree six and Greene's hypergeometric function

Heidi Goodson expressed the number of the rational points on the Dwork hypersurfaces of degree four and of odd degree over the finite field by using Greene's finite field hypergeometric function. Furthermore, Goodson asked about an existence of a formula for the Dwork hypersurfaces of even degree. We extend Goodson's formula to the Dwork hypersurfaces of degree six. Furthermore, our formula is a higher-dimensional and a finite field analogue of Matsumoto-Terasoma-Yamazaki's formula.

9月11日(金)

10:00 – 11:00 浅山 拓哉 (東京工業大学) / Takuya Asayama (Tokyo Institute of Technology)

大きな代数拡大体上の *Drinfeld* 加群の捩れ点

Torsion points of Drinfeld modules over large algebraic extensions

Fix a finitely generated field K over its prime field. For each e -tuple $\sigma = (\sigma_1, \sigma_2, \dots, \sigma_e) \in \text{Gal}(K^{\text{sep}}/K)^e$, let $\tilde{K}(\sigma)$ be the fixed field by σ in the algebraic closure \tilde{K} of K . Geyer and Jarden studied the torsion parts of elliptic curves over $\tilde{K}(\sigma)$ and obtained some finiteness or infiniteness theorems for the torsion subgroups. In this talk, I discuss a Drinfeld module analogue of Geyer-Jarden's results and give some partial answers analogous to them.

11:15 – 12:15 石井 竣 (京都大学) / Shun Ichii (Kyoto University)

On a variant of the uniform boundedness conjecture for Drinfeld modules

Drinfeld modules are function field analogues of abelian varieties. Based on this analogy, Poonen conjectured the uniform boundedness conjecture for torsion of Drinfeld modules. In this talk, we prove a result on the uniform boundedness conjecture for 1-dimensional families of Drinfeld modules over finitely generated fields of generic characteristic. This is an analogue of Cadoret-Tamagawa's result on the p -primary uniform boundedness conjecture for 1-dimensional families of abelian varieties.

13:30 – 14:30 竹内 大智 (東京大学) / Daichi Takeuchi (The University of Tokyo)

Symmetric bilinear forms and Milnor formula for local epsilon factors

Let $f: X \rightarrow \mathbb{A}_k^1$ be a morphism from a smooth variety to an affine line with an isolated singular point. For such a singularity, we have two invariants. One is a non-degenerate symmetric bilinear form (de-Rham), and the other is the vanishing cycles complex (étale).

In this talk, I will give a formula which expresses the local epsilon factor of the vanishing cycles complex in terms of the bilinear form. In particular, the sign of the local epsilon factor is determined by the discriminant of the bilinear form. This can be regarded as a refinement of Milnor formula, which compares the dimension of the bilinear form and the total dimension of the vanishing cycles.

In characteristic 2, we find a generalization of Arf invariant, which is an invariant for an ordinary quadratic singularity, to a general isolated singularity.

14:45 – 15:45 奥村 喜晶 (東京工業大学) / Yoshiaki Okumura (Tokyo Institute of Technology)
On congruence of Galois representations attached to A -motives

Let K/Q be a finite extension of global function fields over finite fields and \mathfrak{p} a finite place of Q . Let G_K be the absolute Galois group of K . Now two \mathfrak{p} -adic G_K -representations V and V' are said to be congruence if associated residue representations are isomorphic in a suitable sense. Then it is natural to discuss when V and V' themselves are isomorphic in a suitable sense. In this talk we consider this problem for Galois representations coming from mixed A -motives over K , which are introduced by Anderson as a higher-dimensional analogue of Drinfeld A -modules. Let v and u be places of K with $v \nmid \mathfrak{p}$ and $u \mid \mathfrak{p}$. Using the theory of equal-characteristic crystalline representations, for sufficiently large \mathfrak{p} , we prove that congruent V and V' are isomorphic on G_{K_v} in a suitable sense if they come from mixed A -motives over K with some reduction conditions (with respect to v and u) and bounded Hodge-Pink weights. This is an analogue of the work of Ozeki and Taguchi on congruence of ℓ -adic representations.

16:00 – 17:00 角濱 寛隆 (京都大学) / Kakuham Hirota (Kyoto University)
Formal degree and Theta correspondence: quaternion cases

By the formal degree conjecture, the behavior of formal degree under the theta correspondence (with a certain rank condition) is expected to be described by a special value of standard γ -factor and some constants. Gan and Ichino has calculated the behavior except for the quaternion cases. In this talk, I extend their work to the quaternion cases. The main ingredient is a precise analyzation of the local Siegel-Weil formula.