第２１回仙台広島整数論集会 アブストラクト

2022 年 7 月 12 日（火）～7 月 15 日（金）
東北大学大学院理学研究科・数理科学記念館
ハイブリッド開催（対面参加は講演者のみ）

7 月 12 日（火）

9:50 – 10:40　高梨　悠吾（東京大学）　/ Yugo Takanashi (University of Tokyo)

p-進体上の GL_n の内部形式の共役自己双対表現の偶奇性

Parity of conjugate self-dual representations of inner forms of GL_n over p-adic fields

There are two parametrizations of discrete series representations of GL_n over p-adic fields. One is the local Langlands correspondence, and the other is the local Jacquet–Langlands correspondence. The composite of these two maps the discrete series representations of an inner form of GL_n to Galois representations called discrete L-parameters. On the other hand, we can define the parity for each selfdual representation depending on whether the representation is orthogonal or symplectic. The composite preserves the notion of self-duality, and it transforms the parity in a nontrivial manner. Prasad and Ramakrishnan computed the transformation law, and Mieda proved its conjugate self-dual analog under some conditions on groups and representations. We will talk about the proof of the general case of this analog. We use the globalization method, as in the proof of Prasad and Ramakrishnan.

10:55 – 11:45　大橋　亮　（横浜国立大学）/ Ryo Ohashi (Yokohama National University)

On the maximality of genus-3 Howe curves

It is well-known that all maximal or minimal curves defined over $\mathbb{F}_{p^2}$ are superspecial, while superspecial curves defined over $\mathbb{F}_{p^2}$ are not necessarily maximal nor minimal. In this paper, we study the maximality or minimality of Howe curves of genus 3. A Howe curve is defined as the desingularization of the fiber product over $\mathbb{P}^1$ of two elliptic curves. We will show that if a Howe curve of genus 3 is superspecial, its standard form is maximal or minimal over $\mathbb{F}_{p^2}$ without taking its $\mathbb{F}_{p^2}$-form.

13:45 – 14:35　田島　凌太（九州大学）　/ Ryota Tajima (Kyushu University)

Shadow が虚数乗法を持つモックモジュラー形式の p 進的な性質について

A p-adic property of mock modular forms whose shadows have complex multiplication

Let f be a mock modular form and g denote the shadow of f. Then there exists a unique p-adic number α and we can obtain a p-adic modular form from f and α. When g has complex multiplication by K and p is split in K, it is known that α is not zero. On the other hand, we don't know much about α for an inert prime p. The speaker proved that α is not zero when p is inert in K under certain conditions. In this talk, I will explain the result.

14:50 – 15:40　中山　裕大（東京大学）　/ Yuta Nakayama (University of Tokyo)

The canonicity of the integral models of RSZ Shimura varieties

Recently, Pappas formulated canonicity, namely universal properties to characterize integral models of Hodge type Shimura varieties defined by Kisin and himself. The formulation modifies more classical canonicity of Milne, and applies to the cases of parahoric levels. An example of Hodge type Shimura varieties was introduced by Rapoport, Smithling and Zhang. They have conjectured relationship, such as a version of Arithmetic Gan–Gross–Prasad conjecture, of automorphic representation theory and the intersection theory on the Shimura varieties and on their moduli theoretic integral modes. We prove that these integral models satisfy the canonicity of Pappas, becoming isomorphic to Kisin–Pappas models as a result.

16:00 – 16:50　前田　洋太（京都大学・ソニーグループ）　/
Yota Maeda (Kyoto University・Sony Group Corporation)

On the geometry of higher dimensional ball quotients

Determination of the birational types of modular varieties is an important problem. In this talk, I will compute the Hirzebruch–Mumford volume of unitary groups and prove that, as a consequence, when the discriminant of imaginary quadratic fields or the rank of unitary groups is sufficiently large, the line bundle, whose sections are modular forms vanishing on branch divisors, is big on ball quotients. As an application, under this condition, I show that ball quotients are of general type, assuming that there exists a low-weight cusp form. As background, modular varieties, especially Siegel modular varieties, by Tai, Freitag and Mumford, and orthogonal modular varieties, by Gritsenko–Hulek–Sankaran and Ma, are known to be of general type when their dimension is large enough, and this result is their unitary analog.

17:05 – 17:55　ミュレー　ジョゼフ（Université Sorbonne Paris Nord・東京大学）　/
Joseph Muller (Université Sorbonne Paris Nord・University of Tokyo)

On the cohomology of the unramified PEL unitary Rapoport-Zink space of signature $(1, n-1)$

Rapoport-Zink (RZ) spaces are moduli spaces which classify the deformations of a p-divisible group with additional structures. It is equipped with compatible actions of p-adic and Galois groups, and their cohomology is believed to play a role in the local Langlands program. So far, the cohomology of RZ spaces is entirely known only in the cases of the Lubin–Tate tower and of the Drinfeld space; in particular both of them are RZ spaces of EL type. In this talk, we consider the unramified PEL unitary RZ space with signature $(1, n-1)$. In 2011, Vollaard and Wedhorn proved that it is stratified by generalized Deligne–Lusztig varieties, whose incidence relations mimic the combinatorics of the Bruhat–Tits building of a unitary group. We compute the cohomology of these strata and we draw some consequences on the cohomology of the RZ space at hyperspecial level. In particular, we prove that it is not admissible in general. When $n = 3, 4$ we deduce an automorphic description of the cohomology of the basic stratum in the corresponding Shimura variety via p-adic uniformization.

7 月 13 日（水）

9:30 – 10:20　高　潤宣　（名古屋大学）　/ Runxuan Gao (Nagoya University)

A Zariski dense exceptional set in Manin's Conjecture: Dimension 2

Manin's Conjecture predicts that the asymptotic behavior of rational points on Fano varieties is controlled by its geometric invariants. The exceptional set in Manin's conjecture was supposed to be a proper closed subset under the Zariski topology and it has been proved for several special families of varieties, such as toric varieties. But counterexamples were discovered then, so the conjecture was modified to assume the exceptional set to be a thin set. Nevertheless, all of these counterexamples are higher dimensional.

Based on a conjectural construction of the exceptional set by Lehmann, Sengupta, and Tanimoto, we constructed a del Pezzo surface with a Zariski dense exceptional set, which gave a negative answer to a question suggested by Lehmann and Tanimoto. As a consequence, we obtained the first counterexample to the original version of Manin's Conjecture in dimension 2.

10:30 – 11:20　肖　歓　（熊本大学）　/ Huan Xian (Kumamoto University)

Campana points of the Heisenberg group

Recently Pieropan, Smeets, Tanimoto and Varilly–Alvarado introduced a log Manin conjecture for Campana points on Fano log pairs and they proved the log Manin conjecture for vector group compactifications. In this talk we consider Campana points on biequivariant compactications of the Heisenberg group and confirm the log Manin conjecture in this case.

11:35 – 12:25　松田　光智（東京大学）　/ Koji Matsuda (Tokyo University)

Torsion points of elliptic curves over cyclotomic fields

The Mordell–Weil group of an elliptic curve over an algebraic number field is a finitely generated abelian group, hence its torsion subgroup is finite. It is easy to compute the torsion subgroup of the Mordell–Weil group of individual given elliptic curve. Conversely for given finite abelian group, it is a hard problem to determine if it is the torsion subgroup of the Mordell–Weil group of an elliptic curve or not. In 1977 Mazur solved this problem over $\mathbb{Q}$ by studying arithmetic properties of modular curves. Roughly speaking, modular curves are algebraic curves parametrizing elliptic curves endowed with certain extra structure. In this talk, we determine all modular curves whose Jacobian varieties have Mordell–Weil rank zero among those defined over cyclotomic number fields, not only over the rational number field, and using it we show a certain classification result of possible torsion subgroups of elliptic curves over cyclotomic fields.

14:00 – 14:50　小泉　淳之介（東京大学）　/ Junnosuke Koizumi (University of Tokyo)

A motivic version of the Hasse–Arf theorem

As one direction to generalize Voevodsky's theory of $\mathbb{A}^1$-invariant sheaves, the theory of reciprocity sheaves was introduced by Kahn–Saito–Yamazaki. One important aspect of the theory is that it gives a unified framework for abelian ramification theory. We generalized the theory of reciprocity sheaves to general base schemes and proved a motivic analogue of the Hasse-Arf theorem for relative curves. This is a joint work with Hiroyasu Miyazaki.

15:05 – 15:55　川邊　大貴（東北大学）　/ Daiki Kawabe (Tohoku University)

種数 1 のファイバー曲面のモチーフ

Chow motives of genus one fibrations

Genus 1 fibrations play a central role in the classification of surfaces over algebraically closed fields. Let $f : X \to C$ be a genus 1 fibration from a surface to a curve, that is, the generic fiber of f has genus 1. Then there exists a Jacobian genus 1 fibration $j : J \to C$ that satisfies some good properties. In this talk, we prove that the motive of X is isomorphic to the motive of J. Using this result, we also prove motivic finite-dimensionality for surfaces not of general type with geometric genus 0.

16:05 – 16:55　宮崎　弘安（NTT 基礎数学研究センタ） /
Hiroyasu Miyazaki (NTT Institute for Fundamental Mathematics)

一般底上のモジュラス付きモチーフ理論について

On motives with modulus over a general base

The theory of motives with modulus, developed in my joint work with Kahn–Saito–Yamazaki, is a generalization of Voevodsky's theory of motives over a field. The aim of this generalization is to get a motivic picture of non-$\mathbb{A}^1$-homotopy invariant phenomena, which were ignored in Voevodsky's theory. In this talk, I will explain the main idea of the theory of motives with modulus, and talk about the recent joint work with Shane Kelly on motives with modulus over a general base.

7 月 14 日（木）

9:30 – 10:20　渡辺　業　（広島大学）　/ Kosaku Watanabe (Hiroshima University)

二進整数環上の二次の無理数についての Collatz 予想の一般化

Generalization of the Collatz conjecture to $\mathbb{Z}_2$

Collatz conjectured that when $n \in \mathbb{N}$, then $T_3^k(n) = 1$ for some $k \in \mathbb{N}$ where T_3 is the Collatz function. In this paper, we study the trajectories of T_p where $p \in \mathbb{Z}_2$ is a quadratic odd number, and $T_p(x) = \frac{px+1}{2}$ if x is odd, and $T_p(x) = \frac{x}{2}$ if x is even. When the defining equation of p over $\mathbb{Z}$ is $p^2 + \alpha p + \beta = 0$, there exist divergent trajectories unless $\alpha \equiv 1 \pmod 2$ and $\beta \equiv 0 \pmod 4$. For p with $p^2 + \alpha p + \beta = 0$ with $\alpha = 1, 3$ and $\beta = 4, 8$, it seems that all trajectories converge to some cycles. And we got some other results.

10:30 – 11:20　臺信　直人（慶應義塾大学）/ Naoto Dainobu (Keio University)

Class groups of the p^n-division fields of elliptic curves and everywhere unramified rational points

Let p be a prime number, n be a positive integer, E be an elliptic curve defined over $\mathbb{Q}$, and $\mathbb{Q}(E[p^n])$ be its p^n-division field. On the p-part of the ideal class group of $\mathbb{Q}(E[p^n])$, several lower bounds are obtained in the literature including recent works by Sairaiji–Yamauchi, Hiranouchi, Ohshita, and Prasad–Shekhar. Their works imply that the class number of the p^n-division field is often divisible by a certain power of p if the Mordell-Weil rank of E is larger than 1.

In this talk, the speaker presents a new result which improves at least partially the above preceding works. This result gives a sufficient condition for the p-divisivility of the class number of the p^n-division field including the case of Mordell–Weil rank 1. Since this condition is very easy to verify in practice, we can apply it to concrete examples as follows.

Let $\{E_{s,t}\}$ be a family of elliptic curves defined by $y^2 = x^3 - (s^4 + t^2)x$ with integers s, t. Then, for every fixed $p \geqslant 5$ and n, we can prove that the above family contains a certain infinite subfamily having the following two novel properties: Their Mordell–Weil ranks are all 1 and the class numbers of their p^n-division fields are all divisible by p^{2n}. The last part of this talk is based on the joint work (arXiv:2205.08946) with Yoshinosuke Hirakawa and Hideki Matsumura.

11:35 – 12:25　村上　和明（東邦大学）　/ Kazuaki Murakami (Toho University)

An analogue of Kida's formula for split prime $\mathbb{Z}_p$-extensions

For a given prime number p, Kida's Riemann–Hurwitz formula describes the variation of the Iwasawa λ-invariants associated with the cyclotomic $\mathbb{Z}_p$-extension in a degree p-extension of CM number fields. In this talk, we consider a non-cyclotomic $\mathbb{Z}_p$-extension and prove a formula which is similar to usual Kida's Riemann–Hurwitz formula. More precisely, let p be an odd prime number and k an imaginary quadratic field in which p splits into $\mathfrak{p}$ and $\mathfrak{p}^*$. There exists a unique $\mathbb{Z}_p$-extension k_∞/k which is unramified outside $\mathfrak{p}$. For an arbitrary abelian extension F over k, we call $F_\infty = Fk_\infty$ the *split prime* $\mathbb{Z}_p$-extension of F. We describe the variation of the $\mathbb{Z}_p$-rank of the $\mathfrak{p}$-ramified Iwasawa modules associated with the split prime $\mathbb{Z}_p$-extension in a degree p-extension L/K which are abelian over k.

14:00 – 14:50　王　沛鐸　（東京大学）　/ Peiduo Wang (University of Tokyo)

On generalized Fuchs theorem over p-adic polyannuli

In this paper, we study finite projective differential modules on p-adic polyannuli satisfying the Robba condition. Christol and Mebkhout proved the decomposition theorem (the p-adic Fuchs theorem) of such differential modules on one dimensional p-adic annuli under certain non-Liouvilleness assumption and Gachet generalized it to higher dimensional cases. On the other hand, Kedlaya proved a generalization of the p-adic Fuchs theorem in one dimensional case. We prove Kedlaya's generalized version of p-adic Fuchs theorem in higher dimensional cases.

keywords: p-adic differential equations; p-adic Fuchs theorem; polyannuli

15:00 – 15:50　三神　雄太郎（東京大学）　/ Yutaro Mikami (University of Tokyo)

Faithfully flat descent of quasi-coherent complexes on rigid analytic varieties over non-archimedean local fields via condensed mathematics

In contrast to scheme theory, it is difficult to construct fundamental theories of rigid geometry such as a theory of quasi-coherent sheaves because of issues of topological algebraic systems such as topological rings and topological modules. Recently Clausen and Scholze have developed a new approach to treat these, which is called condensed mathematics. By using this, we generalize faithfully flat descent of pseudo-coherent complexes on rigid analytic varieties to that of quasi-coherent complexes in the case where the base field is a complete non-archimedean field. In this talk, we will introduce condensed mathematics briefly, then we will explain the difference from fppf-descent in scheme theory and state how it will be resolved.

16:05 – 16:55　根本　裕介（千葉大学）　/ Yusuke Nemoto (Chiba University)

On the K_2-regulator of the Hesse cubic curve and hypergeometric functions

It is known that the Beilinson regulators of certain classes of algebraic varieties are described in terms of generalized hypergeometric functions. In this talk, we discuss the Hesse cubic curve defined by $x_0^3+y_0^3+z_0^3 = 3tx_0y_0z_0$. It is known that its periods are written in terms of Gauss's hypergeometric functions. We construct explicitly some elements in the K_2-group of the Hesse cubic curve by using the tangent lines at the flex points. We compute their Beilinson regulators via the dlog map and express them in terms of generalized hypergeometric functions ${}_3F_2$ and Kampé de Fériet hypergeometric functions. Our elements are integral for some t, and we verify numerically the Beilinson conjecture on the special value of the L-function.

17:05 – 17:55　Peijiang Liu（東京大学）　/ Peijiang Liu (University of Tokyo)

The characteristic cycles of non-confluent ℓ-adic GKZ hypergeometric sheaves

ℓ-adic GKZ hypergeometric sheaves are defined to be étale analogues of GKZ hypergeometric $\mathcal{D}$-modules. We introduce an algorithm of computing the characteristic cycles of certain type of ℓ-adic GKZ hypergeometric sheaves. We compute the irreducible components by a push-forward formula for characteristic cycles of étale sheaves, and compute the multiplicities by considering a comparison theorem between the characteristic cycles of ℓ-adic GKZ hypergeometric sheaves and those of GKZ hypergeometric $\mathcal{D}$-modules. We also explain the limitation of our algorithm.

7 月 **15** 日（金）

9:30 – 10:20　齋藤　耕太（筑波大学）　/ Keita Saito (University of Tsukuba)

Topological properties and algebraic independence of sets of prime-representing constants

Let $(c_k)_{k=1}^{\infty}$ be a sequence of positive integers. We investigate the set of $A > 1$ such that the integer part of $A^{c_1 \cdots c_k}$ is always a prime number for every positive integer k. Let $\mathcal{W}(c_k)$ be this set. The first goal of this talk is to determine the topological structure of $\mathcal{W}(c_k)$. Under some conditions on $(c_k)_{k=1}^{\infty}$, we reveal that $\mathcal{W}(c_k) \cap [0, a]$ is homeomorphic to the Cantor middle third set for some a. The second goal is to propose an algebraically independent subset of $\mathcal{W}(c_k)$ if c_k is rapidly increasing. As a corollary, we disclose that the minimum of $\mathcal{W}(k)$ is transcendental. In addition, we apply the main result to $\mathcal{W}(c_k)$ in the case when $c_1 \cdots c_k = 3^{k!}$. As a consequence, we give an algebraically independent and countably infinite subset of $\mathcal{W}(c_k)$. This research is joint work with Wataru Takeda (Tokyo University of Science).

10:30 – 11:20　加藤　裕基（宇部高専）　/ Yuki Kato (National Institute of Technology, Ube College)

Almost mathematics and its algebraic K-theory from non-unital algebras viewpoint

While almost mathematics has various applications to arithmetic geometry, Quillen mentioned linear algebra over non-unital rings which is the same as almost mathematics introduced by Faltings; and Gabber and Ramero. Quillen's work is more conceptional in the sense of using categorical language: in his work, almost mathematics is characterized as bilocalization of an abelian category of modules. By using the theory of bilocalization, we define the derived category (or stable ∞-category) of almost perfect complexes, enabling us to obtain the K-theory of almost modules, and compare the Thomason–Troubauth K-theory of almost perfect complexes and perfect complex whose weak equivalences are almost quasi-isomorphisms. Furthermore, by modifying K-theory of almost algebra, we prove that the modified K-groups decompose into the almost algebraic K-theory and the almost acyclic part of them.

11:35 – 12:25　後藤　新裕（九州大学）　/ Akihiro Goto (Kyushu University)

Congruence relations connecting Tate-Shafarevich groups with Bernoulli-Hurwitz numbers by elliptic Gauss sums in Eisenstein integers case

There are classical congruences between the class number of the imaginary quadratic field $\mathbb{Q}(\sqrt{-p})$ for a rational prime $p > 3$ and a Bernoulli number or an Euler number. Under the BSD conjecture on the 2-parts of the leading term, Onishi obtained an elliptic generalization of these congruences, which gives congruences between the order of the Tate–Shafarevich group of certain elliptic curves with complex multiplication by the Gaussian integers $\mathbb{Z}[\sqrt{-1}]$ and Mordell–Weil rank 0, and a coefficient of power series expansion of an elliptic function associated to $\mathbb{Z}[\sqrt{-1}]$. In this paper, we provide Onishi's type congruences for the Eisenstein integers $\mathbb{Z}[\frac{-1+\sqrt{-3}}{2}]$.

14:00 – 14:50　任　鑫　（関西大学）　/ Ren Xin (Kansai University)

On q-deformation of some real numbers

The q-real numbers was introduced by S. Morier-Genoud and V. Ovsienko in 2019. In this talk, we focus on q-metallic numbers and q-rational sequences which converge to q-metallic numbers. We found a recurrence formula of the sequence that emerged from the coefficient of q-rational numbers and q-metallic numbers. We consider the radiuses of convergence of them when we assume that q is a complex number. We give an estimation of radiuses of convergence of them, and we solve on conjecture of the lower bound expected which is introduced by L. Leclere, S. Morier-Genoud, V. Ovsienko, and A. Veselov for the metallic numbers and its convergence rational sequence.

15:00 – 15:50　長町　一平（京都大学数理解析研究所）/ Ippei Nagamachi (RIMS, Kyoto University)

Minimal log regular models of hyperbolic curves over discrete valuation rings

Deligne and Mumford proved that a proper hyperbolic curve over a discrete valuation ring R has stable reduction if and only if the Jacobian variety of the curve has stable reduction in the case where the residue field k of R is algebraically closed. In the proof, the theory of minimal regular models played an important role. In this talk, we establish a theory of minimal log regular models of curves. As a key tool for this theory, we introduce a notion of "log blow-down" and give a scheme-theoretic characterization of 2-dimensional log regular local log schemes. Moreover, as an application of this theory, we prove the above equivalence without the assumption on k.