第２３回仙台広島整数論集会 アブストラクト

2024 年 7 月 9 日（火）～7 月 12 日（金）
東北大学大学院理学研究科（青葉山北キャンパス）

7 月 **9** 日（火）

10:00 – 10:45　小原　和馬（東京大学）　/ Kazuma Ohara (University of Tokyo)

Types for Bernstein blocks and their Hecke algebras

The category of smooth complex representations of p-adic groups decomposes into a product of full subcategories, called Bernstein blocks. In this talk, I will explain the result that under a mild condition, every block is equivalent to a depth-zero block, that is closely related with the representation theory of finite reductive groups and much better understood than general blocks. This result is obtained by using the theory of types and an isomorphism of Hecke algebras. This is a joint work with Jeffrey Adler, Jessica Fintzen, and Manish Mishra.

11:00 – 11:45　大江　亮輔（東京大学）　/ Ryosuke Ooe (University of Tokyo)

F-characteristic cycle of a rank one sheaf on an arithmetic surface

The characteristic cycle of a constructible sheaf on a smooth variety was defined on the cotangent bundle by T. Saito. To consider the characteristic cycle in the mixed characteristic case, as a replacement of the cotangent bundle, Saito defined the FW-cotangent bundle of a regular flat scheme over a discrete valuation ring of mixed characteristic. In this talk, we define the F-characteristic cycle satisfying a conductor formula of a rank one sheaf on an arithmetic surface on the FW-cotangent bundle. The definition is based on the computation of the characteristic cycle in the geometric case by Yatagawa. We discuss some properties of the characteristic form, which are necessary for the definition of the F-characteristic cycle.

12:00 – 12:45　張　鑫垚　（東京大学）　/ Xinyao Zhang (University of Tokyo)

The pro-modularity in the residually reducible case

For a continuous odd two dimensional Galois representation over a finite field of characteristic p, it is conjectured that its universal deformation ring is isomorphic to some p-adic big Hecke algebra, called the big R=T theorem. Recently, Deo explored the residually reducible case and proved a big R=T theorem for Q under the assumption of the cyclicity of some cohomology group. However, his method is unavailable for totally real fields since the assumption does not hold any longer. In this talk, we follow the strategy of the work from Skinner-Wiles and Pan on the Fontaine-Mazur conjecture and give a pro-modularity result for some totally real fields, which is an analogue to the big R=T theorem.

14:00 – 14:45　桝澤　海斗（東京大学）　/ Kaito Masuzawa (University of Tokyo)

On the correspondence of simple supercuspidal representations of GSp_{2n} and its inner form

Let F be a nonarchimedean local field. The local Jacquet-Langlands correspondence is the one-to-one correspondence of essential square integrable representations of $\mathrm{GL}_n(F)$ and its inner form which satisfies the character relation. Moreover, It is known that this correspondence preserves the simple supercuspidality. In this talk, we will illustrate that the simple supercuspidality is preserved by a similar correspondence of representations of $\mathrm{GSp}_{2n}(F)$ and its inner form. Once we assume that for each simple supercuspidal representation of $\mathrm{GSp}_{2n}(F)$, there exists a unique irreducible representation of its inner form which satisfies the character relation, then we can see that the corresponding representation is also simple supercuspidal. In addition, we will describe the correspondence of simple supercuspidal representations explicitly.

15:00 – 15:45　田島　凌太（九州大学）　/ Ryota Tajima (Kyushu University)

The p-adic constant for mock modular forms and the degree of a certain modular parameterization of CM elliptic curves

Let $g \in S_k(\Gamma_0(N))$ be a normalized newform and F be a harmonic Maass form that is good for g. Then there exists a unique p-adic constant α_g and we obtained a p-adic modular form from α_g and the holomorphic part F^+. When g has a complex multiplication by an imaginary quadratic field K and $p \nmid N$ is split in $\mathcal{O}_K$, it is known that $\alpha_g = 0$. On the other hand, the speaker showed that α_g is a p-adic unit for an inert prime $p \nmid 2N$ when $\dim_{\mathbb{C}} S_k(\Gamma_0(N)) = 1$. In this talk, we consider a weight 2 normalized CM form g with rational integer Fourier coefficients and E denotes an elliptic curve corresponding to g. Then, there exists a modular parameterization of E associated to the period lattice of g. We denote the degree of this map by C_g. The speaker proved that $C_g\alpha_g$ is a p-adic unit for almost all inert prime p. In this talk, I will explain this result.

16:00 – 16:45　高谷　悠太（東京大学）　/ Yuta Takaya (University of Tokyo)

Moduli spaces of level structure on mixed characteristic local shtukas

Shimura varieties are of central interest in arithmetic geometry. Recently, Pappas and Rapoport introduced axiomatic characterization of canonical integral models of Shimura varieties. In this characterization, local shtukas play an important role as counterparts of p-divisible groups uniformly applicable to any Shimura datum. In this talk, I will explain the relation of canonical integral models at different parahoric levels in terms of local shtukas, restricting to those under hyperspecial levels. I will give a representability criterion of v-sheaves, which is similar in spirit to Artin's theorem on dilatations, and use it to show that canonical integral models under hyperspecial levels represent the flat moduli spaces of level structures over those at hyperspecial levels.

17:00 – 17:45　島田　了輔（香港大学）　/ Ryosuke Shimada (University of Hong Kong)

Basic loci of positive Coxeter type

The notion of affine Deligne-Lusztig variety (ADLV) was first introduced by Rapoport, which has been applied to number theory such as the study of Shimura varieties and a realization of the local Langlands correspondence. Many of these applications make use of the special cases where the ADLV admits a simple description. One large class of such cases is the ADLV of Coxeter type, which has been already classified by Görtz-He-Nie. However, many people (Chan-Ivanov, Howard-Fox-Imai, Trentin,...) have found examples which are not of Coxeter type but admit a simple description. In this talk, I will explain about recent progress on this kind of new examples, including my recent work for GL_n and GSp_6.

7 月 **10** 日（水）

9:00 –　9:45　吉田　裕哉（名古屋工業大学）　/ Yuuya Yoshida (Nagoya Institute of Technology)

Asymptotic and non-asymptotic results for a binary additive problem involving Piatetski–Shapiro numbers

For all $\alpha_1, \alpha_2 \in (1,2)$ with $1/\alpha_1 + 1/\alpha_2 > 5/3$, we show that the number of pairs (n_1, n_2) of positive integers with $N = \lfloor n_1^{\alpha_1} \rfloor + \lfloor n_2^{\alpha_2} \rfloor$ is equal to $\Gamma(1+1/\alpha_1)\Gamma(1+1/\alpha_2)\Gamma(1/\alpha_1+1/\alpha_2)^{-1}N^{1/\alpha_1+1/\alpha_2-1} + o(N^{1/\alpha_1+1/\alpha_2-1})$ as $N \to \infty$, where Γ denotes the gamma function. Moreover, we show a non-asymptotic result for the same counting problem when $\alpha_1, \alpha_2 \in (1,2)$ lie in a larger range than the above. Finally, we give some asymptotic formulas for similar counting problems in a heuristic way.

10:00 – 10:45　竹平　航平（東北大学）　/ Takehira Kohei (Tohoku University)

力学系における放物パラメータの数論的性質

Arithmetic properties of parabolic parameters in dynamical systems

The iteration of polynomial compositions is not only a major subject of study in dynamical systems but also a rich source of intriguing problems in number theory. In dynamical systems theory, the concept known as the parabolic parameter is crucial. This concept is related to bifurcation phenomena in dynamical systems and can be defined for one-parameter families of polynomials. The parabolic parameter is also connected to the famous Mandelbrot set, and there are various studies from the perspective of dynamical systems. However, many aspects of its number-theoretic properties remain unknown. In this talk, I will present the results obtained on the number-theoretic properties of the parabolic parameter for various families of polynomials, focusing particularly on its integrality and height estimates, as well as their applications. This research is a joint work with Kaoru Sano (NTT) and Yuya Murakami (Kyushu university).

11:00 – 11:45　中井　啓太（名古屋大学）　/ Keita Nakai (Nagoya University)

Joint value distribution of the Riemann zeta-function with general shifts

In 1975, Voronin proved the universality theorem for the Riemann zeta-function. Roughly speaking, the universality theorem states that any non-vanishing holomorphic function can be approximated uniformly by a certain shift of the Riemann zeta-function. In 2023, Laurinčikas proposed that whether the universality theorem for the Riemann zeta-function shifted by an exponential function holds or not. This problem was solved for more general shifts by Andersson et al. in 2024. In this talk, we will generalize these results for the joint universality theorem for the Riemann zeta-function with general shifts using a different approach from their method.

12:00 – 12:45　松村　英樹（東京都立大学）/ Hideki Matsumura (Tokyo Metropolitan University)

Quadrature formulas for Bessel polynomials

A quadrature formula is a formula evaluating a definite integration on a given path by a weighted average of function values at finitely many given points (nodes). Examining the existence of certain rational quadratures (quadratures with rational nodes) leads to applications to interesting problems such as the construction of the Hilbert identity in Waring's problem and spherical designs in algebraic combinatorics. In the first half of this talk, we describe the existence theorem and the non-existence theorem of quadratures for the weight function of Bessel polynomials. These are extensions of the results by Sawa–Uchida for classical orthogonal polynomials such as Hermite polynomials to number fields. In the latter half of this talk, we describe recent works on the relationships with polynomials that appear in combinatorics (Narayana polynomials) and a Diophantine problem (Prouhet–Tarry–Escott problem). The latter half is a joint work with Sawa Masanori (Kobe University).

14:00 – 14:45　Lucas Hiroyuki Ragni Hamada（東京工業大学 / Tokyo Institute of Technology）

Classification of torsion subgroups of elliptic curves with rational j-invariant

Since 1997, when B. Mazur successfully classified, up to isomorphism, the torsion subgroups of rational elliptic curves, the classification of torsion subgroups of several families of elliptic curves has been determined. For instance, S. Kamienny, M.A. Kenku and F. Momose classified, up to isomorphism, the torsion subgroups of elliptic curves defined over quadratic extensions of $\mathbb{Q}$. In this talk, after introducing some of these previous results, I will discuss the classification, up to isomorphism, of the torsion subgroups of elliptic curves defined over the field $F := \mathbb{Q}(\sqrt{m} : \ m \in \mathbb{Z})$, with rational j-invariant.

15:00 – 15:45　伊藤　陸統（名古屋大学）　/ Rikuto Ito (Nagoya University)

On the Galois representations associated to CM K3 surfaces and their descriptions by algebraic Hecke characters

The main theorem of complex multiplications for K3 surfaces by Rizov is the analogy of complex multiplication theory for abelian varieties by Shimura and Taniyama. In this talk, we will consider transcendental parts of Galois representations associated with CM K3 surfaces and describe them by algebraic Hecke characters by applying Rizov's result. Also, we will give explicit Hecke characters for some CM K3 surfaces (they have CM by imaginary quadratic fields or cyclotomic fields). They are Skorobogatov's results. Time permitting, we will discuss Kumer surfaces with CM.

16:00 – 16:45　Stefan Reppen（東京大学 / University of Tokyo）

An Ogus Principle for zip period maps

In joint work with W. Goldring, we construct Hasse invariants on the stack of G-zips for any triple (G, μ, r) consisting of a connected reductive $\mathbb{F}_p$-group G, a cocharacter μ and a representation r of G. We define a notion of conjugate line position which generalises the notion of a-number for abelian varieties to such triples. For large classes of groups (all classical types) we compute the vanishing order of the Hasse invariant and show that it agrees with the conjugate line position. We deduce the corresponding result for the special fibre of Hilbert modular varieties, Siegel varieties and orthogonal varieties, all of arbitrary dimension, as well as certain unitary Shimura varieties. These results are analogous to Ogus' on families of Calabi-Yau varieties, and we also recover his result for K3-surfaces. In the talk I will present this work.

17:00 – 17:45　根岸　崚　（北海道大学）　/ Ryo Negishi (Hokkaido University)

Picard–Fuchs equations of the generalized Dwork family

Dwork studied the deformation theory of the zeta function of nonsingular projective hypersurfaces over finite fields by focusing on the Picard-Fuchs equation. In particular, he studied the family now called the Dwork family. In this talk, we derive the Picard-Fuchs equation of the generalized Dwork family introduced by Katz and present a method for computing Katz's deformation matrix using the Picard-Fuchs equation.

7 月 11 日（木）

9:00 – 9:45　渡部　匠　（東京大学）　/ Takumi Watanabe (University of Tokyo)

On the (φ, Γ)-modules corresponding to crystalline representations

From the 1980s to the 1990s, J.-M. Fontaine constructed an equivalence of categories between the category of p-adic Galois representations and the category of (φ, Γ)-modules. After recalling it, I will present my result on the (φ, Γ)-modules corresponding to crystalline representations. This can be seen, in a sense, as a generalization of Wach modules in the ramified case.

10:00 – 10:45　三神　雄太郎（東京大学）　/ Yutaro Mikami (University of Tokyo)

Finiteness of (φ, Γ)-cohomology

In this talk, we present a new proof of finiteness of cohomology of families of (φ, Γ)-modules proved by Kedlaya-Pottharst-Xiao. Our approach is based on condensed mathematics introduced by Clausen-Scholze. This method allows us to handle more general coefficient rings of cohomology. We will also explain the relationship between this study and categorical p-adic Langlands correspondence for locally analytic representations.

11:00 – 11:45　石塚　伶　（東京工業大学）/ Ryo Ishizuka (Tokyo Institute of Technology)

Prismatic approach to a mixed characteristic Kunz's theorem

Kunz's theorem states that a Noetherian ring R of positive characteristic is regular if and only if the Frobenius map on R is (faithfully) flat. This theorem is a fundamental result in the study of singularities in positive characteristic. After reviewing previous research, we will discuss our mixed characteristic variant

of this theorem, utilizing prismatic cohomology, a new p-adic cohomology theory introduced by Bhatt and Scholze. This variant is based on Bhatt's idea that a "mixed characteristic Frobenius map" is the Frobenius lift of prismatic cohomology.

12:00 – 12:45　井上　絢太郎（京都大学）　/ Kentaro Inoue (Kyoto University)

Log prismatic Dieudonné theory for log p-divisible groups

Bhatt and Scholze introduced a new integral p-adic cohomology theory called prismatic cohomology, and Anschutz-Le Bras established a prismatic version of Dieudonné theory. In this talk, we talk about generalization to log p-divisible groups in the framework of log prismatic cohomology established by Koshikawa and Yao. If time permits, we will explain the application to Shimura varieties.

14:00 – 14:45　田中　拓弥（東京工業大学）/ Takuya Tanaka (Tokyo Institute of Technology)

Modulo p of Hecke L values over imaginary quadratic fields

The non-vanishing modulo p property for the algebraic part of the Hecke L values $L^{\mathrm{alg}}(\chi)$ is very important. Lamplugh, and Lei-Kundu in 2023 established a theory of elliptic function measure, which is a measure associated to elliptic curves with complex multiplications and obtained some mod p properties of $L^{\mathrm{alg}}(\chi\psi^{-1})$ where χ: critical Hecke character of imaginary quadratic field , ψ: finite character with l-power conductor. We partially generalized their results and obtained non-vanishing modulo p property of $L^{\mathrm{alg}}(\chi\psi^{-1})$ for more wide class of ψ. Our method also provides some modulo p properties of abelian functions for CM-type abelian varieties.

15:00 – 15:45　後藤　新裕（九州大学）　/ Akihiro Goto (Kyushu University)

On some values which do not belong to the image of Ramanujan's tau function

In 1947, Lehmer conjectured that Ramanujan's τ-function never vanishes, where the function $\tau : \mathbb{Z}_{>0} \longrightarrow \mathbb{Z}$ is defined by $x\prod_{n\geq 1}(1-x^n)^{24} = \sum_{n\geq 1}\tau(n)x^n$. This conjecture is still open, althogh it is known by Serre that the set of prime numbers p such that $\tau(p) = 0$ has density 0 within the primes. It is natural to consider whether any given integer belongs to the image of τ-function. Recently, it is proved that $\pm\ell, \pm 2\ell, \pm 2\ell^2$ are not τ-values, where $\ell < 100$ is an odd prime by Balakrishnan, Ono, Craig and many other people. The speaker proved that $\pm\ell, \pm 2\ell, \pm 4\ell, \pm 8\ell$ are not τ-values except for 14 cases, where $\ell < 1000$ is an odd prime. In my talk, the speaker will remark on the relation between my result and Atkin–Serre conjecture, which states on the asymptotic behavior of $\tau(p)$.

16:00 – 16:45　菅野　隼　（東北大学）　/ Hayato Kanno (Tohoku University)

レベル付き多重 Eisenstein 級数のシャッフル正規化

Shuffle regularization for multiple Eisenstein series of arbitrary level

Multiple Eisenstein series (MES) is a holomorphic function on the upper half plane, which is a generalization of double Eisenstein series introduced by Gangl, Kaneko and Zagier. As a connection to multiple zeta value, Bachmann and Tasaka showed that the Fourier expansion of MES is obtained from the coproduct of formal iterated integrals corresponding to multiple zeta values. They also constructed shuffle regularized MES. For general level, Yuan and Zhao studied double Eisenstein series of level N and obtained analogous results of Gangl, Kaneko and Zagier. In this talk, we introduce MES of level N and expand the results of Bachmann and Tasaka for arbitrary level.

17:00 – 17:45　ヤダデン　ハレフ（名古屋大学）/ Khalef Yaddaden (Nagoya University)

The torsor of double shuffle among cyclotomic multiple zeta values and de Rham and Betti coproducts stabilizers

Racinet described the double shuffle and regularization relations between multiple polylogarithm values at Nth roots of unity via a $\mathbb{Q}$-scheme DMR^{ι} where $\iota : G \hookrightarrow \mathbb{C}^{\times}$ is a group embedding from a finite cyclic group G of order N to $\mathbb{C}^{\times}$. Then, Enriquez and Furusho proved, when $N = 1$, that a subscheme $\mathsf{DMR}^{\iota}_{\times}$

is a torsor of isomorphisms between Betti and de Rham objects. In this talk, we establish a cyclotomic generalization of this result ($N \geq 1$). First, we explicit the torsor structure of $\mathsf{DMR}^{\iota}_{\times}$. Then, we introduce in this context the adequate de Rham and Betti objects : the former arise from a crossed product algebra and enables a reformulation of Racinet's harmonic coproduct closer to the formalism introduced by Enriquez and Furusho; the latter, on the other hand, arise from a group algebra of the orbifold fundamental group $\left(\mathbb{C}^{\times} \smallsetminus \mu_N\right)/\mu_N$, where μ_N is the group of Nth roots of unity. Finally, we show the existence of a coalgebra and Hopf algebra coproduct such that $\mathsf{DMR}^{\iota}_{\times}$ is a torsor of isomorphisms relying these Betti coproducts to their de Rham counterparts.

7 月 12 日（金）

9:00 – 9:45　山田　智宏（大阪大学）　/ Tomohiro Yamada (Osaka University)

Integers whose sum of divisors is prime power

We study arithmetic properties of integers N such that $\sigma(N) = p^e$ for some prime p. Factor $N = q_1^{f_1} \cdots q_r^{f_r}$ with $q_1 < \cdots < q_r$ prime and put $N_1 = N/q_1^{f_1}$ if $q_1 < 10^5$ and $N_1 = N$ otherwise. Combining lower bounds for linear forms of logarithms of rational numbers with known results for Nagell-Ljunggren equation and Goormaghtigh equation, we show that if $\sigma(N) = p^e$, then $\sigma(N_1)/N_1 < 9$.

10:00 – 10:45　Philip Séverin（京都大学 / Kyoto University）

The semi-stability degree for abelian varieties

We will present an effective optimal version of the semi-stable reduction theorem of Grothendieck. First we will introduce the problem of the semi-stability degree starting from the case of elliptic curves and up to the theorem of Grothendieck. We will then present one of the principal objects in this work : the finite monodromy groups of an abelian variety with some of their properties and relation to semi-stable reduction. We will sketch how to obtain a local-global principle for these groups which will reduce our problem to a purely local one. The last part of the talk will be dedicated to the construction of abelian varieties with prescribed finite monodromy groups by deformation and descent starting from polarised abelian varieties over finite fields.

11:00 – 11:45　志賀　明日香（東北大学）　/ Asuka Shiga (Tohoku University)

On behaviors of the 2-torsion subgroup of the Tate–Shafarevich group under quadratic number field extensions

Let E be an elliptic curve over a number field K. The Tate–Shafarevich group $Ш(E/K)$ measures the failure of the local-global principle for genus 1 curves over K. We study behaviors of the 2-torsion subgroup of the Tate–Shafarevich group $Ш(E/K)[2]$ under the quadratic number field extension $L = K(\sqrt{D})/K$. We prove that $\#Ш(E/L)[2]/\#Ш(E_D/K)[2]$ can be made arbitrarily large where E_D/K is the twist of E/K, under the conjecture that for arbitrary elliptic curve A over K, there exists a $D \in K^{\times}$ such that rank of A_D/K is 0. In the direction of decreasing $\#Ш(E/L)[2]$, for certain elliptic curves E over $\mathbb{Q}$, we prove that the order of $\#Ш(E/\mathbb{Q}(\sqrt{D}))[2]$ is less than 4 for infinitely many prime numbers D.

12:00 – 12:45　浅山　拓哉（東京工業大学）/ Takuya Asayama (Tokyo Institute of Technology)

有限生成関数体の大きな代数拡大体上の Drinfeld 加群の捩れについて

On the torsion of Drinfeld modules over large algebraic extensions of finitely generated function fields

For a field K and an e-tuple σ of elements in the absolute Galois group G_K of K, let $\overline{K}(\sigma)$ be the fixed field of σ in the algebraic closure $\overline{K}$ of K. Jacobson and Jarden in their 2001 paper proved some finiteness theorems on the torsion of abelian varieties over $\overline{K}(\sigma)$ when K is a finitely generated field of the field $\mathbb{Q}$ of rational numbers. In this talk, I will discuss the torsion of Drinfeld modules over $\overline{K}(\sigma)$ when K is a finitely generated function field. I will extend my previous work and give complete analogues of Jacobson–Jarden's theorems for Drinfeld modules of generic characteristic. As an application, I will show

the structure theorem for Drinfeld modules over the maximal Galois subextension $\overline{K}[\sigma]$ in $\overline{K}(\sigma)/K$, where $\sigma \in G_K^e$, $e \geq 2$.

14:00 – 14:45　石塚　康介（東北大学）　/ Kosuke Ishizuka (Tohoku University)

An application of the spherical completion to finite-dimensional normed spaces

The isometry classification problem of finite-dimensional normed spaces over a non-spherically complete field is open. In the case of 2-dimensional normed spaces, van Rooij succeeded in classifying normed spaces by using a hole. Holes are maximal chains of closed balls and norms associated with holes are defined by limits. Therefore, holes are complicated and they are not suitable for using in general cases. In this talk, we will use the spherical completion instead of holes. From the perspective of the spherical completion, we can see a hole as a point of a spherically complete field, which is easy to study. As a result, we can classify 3-dimensional normed spaces. Furthermore, we apply the classification to study strictly epicompact sets.

15:00 – 15:45　白石　伝助（東京理科大学）/ Densuke Shiraishi (Tokyo University of Science)

On the ℓ-adic Galois side of polylogarithm functional equations

In this talk, we will discuss functional equations of the ℓ-adic Galois polylogarithm introduced by Z. Wojtkowiak as the ℓ-adic etale analog of the complex polylogarithm for any prime number ℓ. The ℓ-adic Galois polylogarithm originates from the Galois action on the pro-ℓ etale fundamental groupoid of the projective line minus three points. In particular, I would like to focus on the ℓ-adic Galois versions of Landen's 3-terms and Spence-Kummer's 9-terms trilogarithm functional equations, that have some ℓ-adic Galois extra terms. We shall derive these functional equations from algebraic relations (chain rules) between ℓ-adic Galois associators. Finally, I will explain that by reinterpreting the proof after replacing ℓ-adic Galois associators with complex KZ associators, we obtain a new algebraic proof of the complex trilogarithm functional equations. A part of this talk is based on a joint work with Hiroaki Nakamura.