Errors in "Introduction to Infinite Analysis"

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§1. Errors in "Introduction"

Leonhard Euler, who is celebrated for the 300th anniversary of his birth in 2007, continues to influence many researchers with his voluminous works. Among them, "Introduction to Infinite Analysis"(E101, E102), the first part of his famous trilogy on analysis that forms the basis of his works, is an extremely important work.

Trilogy on analysis	the number of volumes	written	published
Introduction to Infinite Analysis	2	1745	1748
Foundations of Differential Calculus	1	1748	1755
Foundations of Integral Calculus	3	1763	1768 $\sim$ 1770

In "Leonhardi Euleri Opera Omnia"[3], it has been pointed out that there are a large number of numerical calculation errors in this work. There are more than 100 such errors, but when we examined these errors, we found that many of them were not necessarily wrong. This is because the complete collection handles the numbers as "rounded off", and if we consider that the numbers are "rounded down", nearly half of the numbers are correct. On the other hand, dozens of new errors that were not recorded in the complete collection were discovered, leaving about 80 errors that cannot be explained by "rounding off" or "truncation". Of course, there is a high possibility that it was a simple calculation error, so we reconsidered each numerical value and calculation method, but in the end we were unable to attribute the cause of these errors to a simple calculation error. If we assume that these errors are not caused by chance, we must consider the following possibility.

Aren't these errors intentional?

In this essay, we will introduce this hypothesis. First, we will answer some questions that most people may have.

Question 1 Would a great mathematician do such a strange thing?
Question 2 Is there any benefit in changing correct values to wrong values?
Question 3 Why hasn't there been a hypothesis like this until now?
Question 4 Why do we propose such a hypothesis?

Answer 1 Euler was invited by Frederick the Great to establish and manage the Academy at the time. The year before the publication of "Introduction", the great musician Johann Sebastian Bach, also known as the Great Bach, famously posed a riddle to King Frederick the Great called "The Musical Offering". One of the dedicated scores cannot be played as it is, because the first half of notes are reversed. The problem was how to play these strange musical scores. Great Bach was already 62 years old at the time, but he had this sense of playfulness.

It is also well known that the great mathematician Newton sent Leibniz a ciphertext using anagrams regarding the flow rate method in nealy same period. It is not unnatural to encrypt results that one would like to refrain from making public, but for which one would like to claim a right of first refusal.

There were great predecessors like those mentioned above, so it wouldn't be so strange that young Euler between the ages of 38 and 41 gave riddles using his specialty in calculations. Please judge whether it is strange or not after considering the numerical data and the hypothesis in this essay.

Answer 2 Even if the numerical values that mathematicians and scientists do not use are slightly wrong, there is no problem practically. Furthermore, it is not necessarily in the best interest of the reader to write down the correct numerical value. This is because, for enthusiastic calculators who recalculate, if correct numbers are written, it only serves as a check. However, if a numerical value is written with some interesting fact included in the error, the calculator who correctly calculates the numerical value will be rewarded with the achievement. In other words, most mathematicians and scientists do not have any practical problems, and there are possibly benefits for calculators who check the values. It would not be unnatural for Euler, a great calculator, to try to provide results to calculators in future generations.

Answer 3 The corrected numbers in Euler's Complete Works are due to rounding, so it is not possible to understand the meaning by examining the errors. Furthermore, as studying errors may lead to criticizing Euler, who was considered a great calculator, such a task may be reluctant to do for scholars who respect Euler. In any case, there has never been any merit in checking errors.

Answer 4 This is because we believe that this hypothesis will advance our understanding of Euler's actions and writings. We also assume that Euler's intentions in including errors were both grand and delicate. We believe that this hypothesis is worth considering, but we must leave the final judgment to the readers.

§2. Lists of Errors

The numerical errors exist only near the last digit and are not considered to be a typographical error. In the list below, after writing down the correct numbers, we wrote down only the incorrect numbers in the "Introduction". (Calculation is according to [7].)

For example, in the list of P0, the correct truncated value with 7 significant digits is $E = 4.216965$, but the Introduction says $E = 4.216964$. Also, the correct cutoff value to 7 digits after the decimal point for $\log_{10} 2$ is 0.3010299, but in the "Introduction" it is written as 0.3010300. Since $\log_{10} 2 = 0.30102999\cdots$, if we round it off, 0.3010300 is more correct. This number is extremely delicate.

Volume 1

P0		$\log_{10} 5$		
$A = 1.000000$		$lA = 0.0000000$		
$B = 10.00000$		$lB = 1.0000000$		$C = \sqrt{AB}$
$C = 3.162277$		$lC = 0.5000000$		$D = \sqrt{BC}$
$D = 5.623413$		$lD = 0.7500000$		$E = \sqrt{CD}$
$E = 4.216965$	4	$lE = 0.6250000$		$F = \sqrt{DE}$
$F = 4.869675$	4	$lF = 0.6875000$		$G = \sqrt{DF}$
$G = 5.232991$		$lG = 0.7187500$		$H = \sqrt{FG}$
$H = 5.048065$		$lH = 0.7031250$		$I = \sqrt{FH}$
$I = 4.958068$	9	$lI = 0.6953125$		$K = \sqrt{HI}$
$K = 5.002864$	5	$lK = 0.6992187$		$L = \sqrt{IK}$
$L = 4.980416$		$lL = 0.6972656$		$M = \sqrt{KL}$
$M = 4.991627$		$lM = 0.6982421$		$N = \sqrt{KM}$
$N = 4.997243$	2	$lN = 0.6987304$		$O = \sqrt{KN}$
$O = 5.000053$	2	$lO = 0.6989746$	5	$P = \sqrt{NO}$
$P = 4.998647$		$lP = 0.6989525$		$Q = \sqrt{OP}$
$Q = 4.999350$		$lQ = 0.6989135$		$R = \sqrt{OQ}$
$R = 4.999701$		$lR = 0.6989440$		$S = \sqrt{OR}$
$S = 4.999877$	6	$lS = 0.6989593$	2	$T = \sqrt{OS}$
$T = 4.999965$	3	$lT = 0.6989669$	8	$V = \sqrt{OT}$
$V = 5.000009$	8	$lV = 0.6989707$		$W = \sqrt{TV}$
$W = 4.999987$	4	$lW = 0.6989688$	7	$X = \sqrt{WV}$
$X = 4.999998$	7	$lX = 0.6989698$	7	$Y = \sqrt{VX}$
$Y = 5.000003$		$lY = 0.6989703$	2	$Z = \sqrt{XY}$
$Z = 5.000000$		$lZ = 0.6989700$		
$\log_{10} 2$	$=$	0.3010299	300	
$1/\log_{10} 2$	$=$	3.3219280	77	

P1	$\log n$	
$l1$	0.0000000000000000000000000	
$l2$	0.6931471805599453094172321	
$l3$	1.0986122886681096913952452	
$l4$	1.3862943611198906188344642	
$l5$	1.6094379124341003746007593	
$l6$	1.7917594692280550008124773	
$l7$	1.9459101490553133051053527	4639
$l8$	2.0794415416798359282516963	4
$l9$	2.1972245773362193827904904	5
$l10$	2.3025850929940456840179914	

P2 $s_k = (-1)^{\frac{k-1}{2}} \dfrac{\pi^k}{2^k k!}$

01	$+1.5707963267948966192313216916$	
03	$-0.6459640975062462536557565638$	6
05	$+0.0796926262461670451205055494$	88
07	$-0.0046817541353186881006854639$	2
09	$+0.0001604411847873598218726608$	5
11	$-0.0000035988432352120853404585$	0
13	$+0.0000000569217292196792681177$	1
15	$-0.0000000006688035109811467232$	24
17	$+0.0000000000060669357311061956$	0
19	$-0.0000000000000437706546731374$	0
21	$+0.0000000000000002571422892860$	56
23	$-0.0000000000000000012538995405$	3
25	$+0.0000000000000000000051564551$	0
27	$-0.0000000000000000000000181239$	
29	$+0.0000000000000000000000000550$	49

$c_k = (-1)^{\frac{k}{2}} \dfrac{\pi^k}{2^k k!}$

00	$+1.0000000000000000000000000000$	
02	$-1.2337005501361698273543113749$	5
04	$+0.2536695079010480136365633663$	59
06	$-0.0208634807633529608730516372$	64
08	$+0.0009192602748394265802417162$	58
10	$-0.0000252020423730606054810530$	26
12	$+0.0000004710874778818171503670$	65
14	$-0.0000000063866030837918522410$	08
16	$+0.0000000000656596311497947236$	0
18	$-0.0000000000005294400200734623$	0
20	$+0.0000000000000034377391790986$	1
22	$-0.0000000000000000183599165215$	2
24	$+0.0000000000000000000820675330$	27
26	$-0.0000000000000000000003115284$	5
28	$+0.0000000000000000000000010167$	5
30	$-0.0000000000000000000000000028$	6

P3 $\dfrac{2(2^{n+1}-1)\pi^n |B_{n+1}|}{(n+1)!} - \dfrac{4}{\pi}$

01	$+0.2975567820597$	
03	$+0.0186886502773$	
05	$+0.0018424752035$	4
07	$+0.0001975800715$	4
09	$+0.0000216977373$	245
11	$+0.0000024011369$	70
13	$+0.0000002664133$	2
15	$+0.0000000295864$	
17	$+0.0000000032867$	
19	$+0.0000000003651$	
21	$+0.0000000000405$	
23	$+0.0000000000045$	
25	$+0.0000000000005$	

$\dfrac{1}{2^{n-1}\pi} - \dfrac{2\pi^n |B_{n+1}|}{(n+1)!}$

01	-0.2052888894145	
03	-0.0065510747882	
05	-0.0003450292553	4
07	-0.0000202791060	
09	-0.0000012366527	
11	-0.0000000764958	9
13	-0.0000000047597	
15	-0.0000000002969	
17	-0.0000000000185	
19	-0.0000000000011	

$$\zeta(k) = \sum_{n=1}^{\infty} \frac{1}{n^k} = 1 + \frac{1}{2^k} + \frac{1}{3^k} + \frac{1}{4^k} + \frac{1}{5^k} + \frac{1}{6^k} + \cdots .$$

$$\zeta(2) = \frac{2^0}{1\cdot 2\cdot 3}\cdot\frac{1}{1}\pi^2 \qquad \zeta(4) = \frac{2^2}{1\cdot 2\cdot 3\cdot 4\cdot 5}\cdot\frac{1}{3}\pi^4$$

$$\zeta(6) = \frac{2^4}{1\cdot 2\cdot 3\cdots 7}\cdot\frac{1}{3}\pi^6 \qquad \zeta(8) = \frac{2^6}{1\cdot 2\cdot 3\cdots 9}\cdot\frac{3}{5}\pi^8$$

$$\zeta(10) = \frac{2^8}{1\cdot 2\cdot 3\cdots 11}\cdot\frac{5}{3}\pi^{10} \qquad \zeta(12) = \frac{2^{10}}{1\cdot 2\cdot 3\cdots 13}\cdot\frac{691}{105}\pi^{12}$$

$$\zeta(14) = \frac{2^{12}}{1\cdot 2\cdot 3\cdots 15}\cdot\frac{35}{1}\pi^{14} \qquad \zeta(16) = \frac{2^{14}}{1\cdot 2\cdot 3\cdots 17}\cdot\frac{3617}{15}\pi^{16}$$

$$\zeta(18) = \frac{2^{16}}{1\cdot 2\cdot 3\cdots 19}\cdot\frac{43867}{21}\pi^{18} \qquad \zeta(20) = \frac{2^{18}}{1\cdot 2\cdot 3\cdots 21}\cdot\frac{1222277}{55}\pi^{20}$$

$$\zeta(22) = \frac{2^{20}}{1\cdot 2\cdot 3\cdots 23}\cdot\frac{854513}{3}\pi^{22} \qquad \zeta(24) = \frac{2^{22}}{1\cdot 2\cdot 3\cdots 25}\cdot\frac{1181820455}{273}\pi^{24}$$

$$\zeta(26) = \frac{2^{24}}{1\cdot 2\cdot 3\cdots 27}\cdot\frac{76977927}{1}\pi^{26}$$

(These values are correct.)

PA $\zeta(k)\left(1-\frac{1}{2^k}\right)$

$A =$	1.23370055013616982735431	
$B =$	1.01467803160419205454625	
$C =$	1.00144707664094212190647	
$D =$	1.00015517902529611930298	
$E =$	1.00001704136304482548818	50816
$F =$	1.00000188584858311957590	
$G =$	1.00000020924051921150010	
$H =$	1.00000002323715737915670	
$I =$	1.00000000258143755665977	
$K =$	1.00000000028680769745558	
$L =$	1.00000000003186677514044	
$M =$	1.00000000000354072294392	
$N =$	1.00000000000039341246691	
$O =$	1.00000000000004371244859	
$P =$	1.00000000000000485693682	
$Q =$	1.00000000000000053965957	
$R =$	1.00000000000000005996217	
$S =$	1.00000000000000000666246	
$T =$	1.00000000000000000074027	
$V =$	1.00000000000000000008225	
$W =$	1.00000000000000000000913	
$X =$	1.00000000000000000000101	

PB $\zeta(k)\frac{1}{2^k}$

$\alpha =$	0.41123351671205660911810	
$\beta =$	0.06764520210694613696975	
$\gamma =$	0.01589598534350701780804	
$\delta =$	0.00392217717264822007570	
$\varepsilon =$	0.00097753376477325984896	8
$\xi =$	0.00024420070472492872273	4
$\eta =$	0.00006103889453949332915	
$\theta =$	0.00001525902225127271503	69977
$\iota =$	0.00000381471182744318008	
$\kappa =$	0.00000095367522617534053	
$\lambda =$	0.00000023841863595259255	154
$\mu =$	0.00000005960464832831555	
$\nu =$	0.00000001490116141589813	
$\zeta =$	0.00000000372529031233986	
$o =$	0.00000000093132257548284	
$\pi =$	0.00000000023283064370808	7
$\rho =$	0.00000000005820766091685	
$\sigma =$	0.00000000001455191522858	
$\tau =$	0.00000000000363797880710	
$\upsilon =$	0.00000000000090949470177	
$\phi =$	0.00000000000022737367545	4
$\chi =$	0.00000000000005684341886	
$\psi =$	0.00000000000001421085471	
$\omega =$	0.00000000000000355271368	7

PC	$\upsilon(k) = \sum_{p:prime} \frac{1}{p^k}$	
02	0.452247420041065	222
04	0.076993139764246	52
06	0.017070086850637	9
08	0.004061405366518	5
10	0.000993603574437	3633
12	0.000246026470035	3
14	0.000061244396725	
16	0.000015282026219	
18	0.000003817278702	
20	0.000000953961124	3
22	0.000000238450446	
24	0.000000059608184	
26	0.000000014901555	
28	0.000000003725333	
30	0.000000000931326	3
32	0.000000000232830	
34	0.000000000058207	
36	0.000000000014551	

Volume 2

$\sqrt{2}$	$= 1.41421356$	1.4142356
$\log_{10} 2^{\sqrt{2}}$	$= 0.4257207$	74
$2^{\sqrt{2}}$	$= 2.665144$	86
$10^{\sqrt{2}}$	$= 25.954553$	5870

$\cos \log 2$	$= 0.76923890136397$	5408
$\log \pi$	$= 1.1447298858494001741434273$	37
$\sin 1$	$= 0.84147098480789$	514
$\cos 1$	$= 0.54030230586813$	4341

$s + \cot s$	
$+0.63661977$	
-0.17200818	7
-0.09062597	6
-0.05892836	4
-0.04258548	3

§3. Summary of hypothesis

The contents of hypothesis [7] can be summarized as follows.

Euler intentionally included errors in the numerical values in “Introduction to Infinite Analysis” and posed the following problem.

Volume 1	7 Lists	Solution 1	Solution 2
P0	$\log_{10} 5$	All problems	All Solutions
P1	Log. values	Prime decomp., Decimal sys.	Alphabet psalms 37, 111, 112
P2	Sine, Cosine	Euler’s formula	Score for the psalm
P3	Tan, Cotan	Unicursal prob., Euler char.	Period of psalms (2/1, 1/2)
PA	Zeta values A	First irregular prime 37	Mercury
PB	Zeta values B	Second 59, (67,101,103,131)	Venus, (Earth, Mars, Comet, Jupiter)
PC	Zeta values C	Third 67, (149,157)	Earth, (Saturn, Comets)
Volume 2	Trans. numbers	Check for solutions	Connection to P0

Below, we will explain the abnormal error of P2 , which is the main problem in the first half . The following points can be immediately gleaned from the numbers.

1. There are errors in 28 out of 31 numbers.
2. The accuracy is mysterious, with 28 digits after the decimal point.
3. The ratios of errors are rapidly increasing.
4. The errors are within the last digit.
5. The absolute value of only one data is greater than the positive value.

It would be difficult to find other numerical data that meet these conditions. However, what is even more unusual is that these errors can be interpreted as follows.

The values of P2 are the coefficient of the Maclaurin expansion of sin x (sine) and cos x (cosine). Strings have been the main element that creates the sound of musical instruments. This reminds us of a musical score, and we associate the error $0123456789\cdots$ with CDEFGABcde$\cdots$. Furthermore, the coefficients are arranged alternately in the order of their power exponents (that is, Euler’s formula). Euler was a devout Christian, and the number of errors and precision of 28 digits suggests the hymn’s 8686 meter (common metre double). Since the number “555” appears in several lists, we include this as three beats, and obtain the score in the next page.

6. This score is extremely skillfully composed.

This is what seems to be the most unusual point. The first, second, and last scale in the common metre double are “C”, that is, the error is 0. The intro “CC GE” starts from the first C and becomes a consonant chord (just intonation) of 1:1 (1:2), 2:3, 4:5, with unison (octave), perfect fifth , and major third. Note that Euler’s temperament was just intonation composed of prime numbers 2, 3, and 5 . As for the three-tone chords, the odd-numbered chords of the seven main chords appear in order: “CEG”, “EGB”, “ GBd”, and finally “Bdf”, making the piece a song that can be called a model for chords.

Additionally, the overall numbers have been nicely adjusted as shown below.

$$\begin{cases} x'_n = (\text{decimal part of nth coefficient given by Euler }) \times 10^{28} \\ x_n = (\text{decimal part of nth coefficient in positive values}) \times 10^{28} \end{cases}$$

$$e_n = x'_n - x_n(error)$$

$$\sum_{n=0}^{30} |e_n| = 111, \quad \sum_{n=0}^{30} e_n = -3 \Rightarrow \frac{\sum_{n=0}^{30} |e_n|}{\left|\sum_{n=0}^{30} e_n\right|} = 37.$$

Detailed analysis by Professor T.

$$\begin{aligned} \sum_{n=0}^{30} |e_n| &= \sum_{n=0}^{8} |e_{2n}| + \sum_{n=1}^{8} |e_{2n-1}| + \sum_{n=17}^{30} |e_n| &= 37 + 37 + 37 \\ &= \sum_{n=0}^{15} |e_{2n}| + \sum_{n=1}^{15} |e_{2n-1}| &= 55 + 56 = 111. \end{aligned}$$

$$\begin{aligned} \sum_{n=0}^{30} x'_n &= \sum_{n=0}^{15} x'_{2n} + \sum_{n=1}^{15} x'_{2n-1} &= 1 + (-1) \\ &= \sum_{n=0}^{30} x_n + \sum_{n=0}^{30} e_n &= 3 + (-3) \quad = 0. \end{aligned}$$

In this way, 37 appears as the solution for P1 and PA, and the total number of beats is 59, which is the solution for PB.

§4. Reasons and refutation methods

The hypothesis helps to explain many strange statements in Euler's work. In other words, the solutions of the problems were shown through a strange description.

Numerical values and phrases in the examples and problems of "Introduction"

Chapter 6 provides example problems such as "Find the value of $2^{\frac{7}{12}}$" and "Assuming that the number of humans increased from 6 people after the flood ...". We think it is appropriate to interpret the former as the perfect fifth of equal temperament that appears in music, and the latter as the flood recorded in the book of Genesis in the Old Testament. Hymns are associated with music and the Bible. It is very likely that the hymns of the Calvinist are found in the Psalms.

The Alphabet of "Introduction"

Euler entered the Faculty of Theology in order to become a pastor. Since Latin, Greek, and Hebrew were required subjects, he had a wealth of knowledge of ancient languages. The list P0 includes 24 Latin letters from the 14th to 16th centuries, excluding J and U. The list B includes all 24 Greek letters. Then, the 22 alphabets in the list A , in which Y and Z are strangely omitted, brings to mind the ancient Hebrew letters. These, together with the psalms mentioned above, brings to mind the alphabetic psalms. Furthermore, the numbers 26 (zeta values are listed up to 26) and 18 in the list C can be similarly interpreted. That is, 26 is the number of Latin letters from the 18th century (letters A to Z), and 18 is the number of Latin letters (letters A to T, excluding G and J) that correspond in order to the Paleo-Hebrew alphabet. The correspondence between the list of zeta values and the above-mentioned writing system is in perfect harmony.

Symbols of Sun and moon in paper E352

The following symbols are used in the paper entitled "On the Beautiful Relationship Between Power Sums and Power Sums of Reciprocals", written in 1749 and published in 1768.

$$\odot \;\cdot\; 1^m - 2^m + 3^m - 4^m + 5^m - 6^m + 7^m - 8^m + \&c.$$

$$\rightmoon \;\cdot\; \frac{1}{1^n} - \frac{1}{2^n} + \frac{1}{3^n} - \frac{1}{4^n} + \frac{1}{5^n} - \frac{1}{6^n} + \frac{1}{7^n} - \frac{1}{8^n} + \&c.$$

These symbols are used to clearly show that the zeta function is compared to celestial bodies such as the sun and the moon. These analogies were so puzzling to us that we began to think that Euler might have posed a problem. It is presumed that behind this parable is the idea that "all things are numbers". Furthermore, Euler observed an annular solar eclipse on July 25, 1748, the year before he wrote the paper, and he might compare this eclipse to a functional equation of the zeta function. Moreover, Euler had arrived in Berlin from Russia on July 25, 1741, exactly seven years before that date, which shows his enthusiasm for this eclipse. In fact, he wrote two papers related to this solar eclipse.

Zeta values up to 34 in paper E352

In the above paper, the zeta values up to 34 are written to "indicate as much as we have calculated so far". It is rare for a mathematician to calculate this much by hand, but it would not be strange if he were interested in irregular prime numbers. To determine whether p is an irregular prime number , check the numerator of the zeta values up to $p-3$. Therefore, in order to determine the smallest irregular prime number 37 , it is sufficient to examine the zeta values up to $34 = 37 - 3$, and it is natural to find the values up to 34. Note that if we only want to determine whether a prime number is an irregular prime number, we can use calculations modulo that prime number, so irregular prime numbers with large exponents are considered to be investigated using this type of calculation.

Structure of the book E343 "Letters to a German Princess"

The strongest evidence lies in this famous book. If we pose a problem, it is natural to write down the solution. This work was written between 1760 and 1762 and published in the same year as the paper E352, which included the symbols for the sun and moon. We believe that the solutions to the problems were presented in order in the first of the three volumes. The following is a summary of the contents of Volume 1, with parts that are assumed to correspond to the solutions shown in bold.

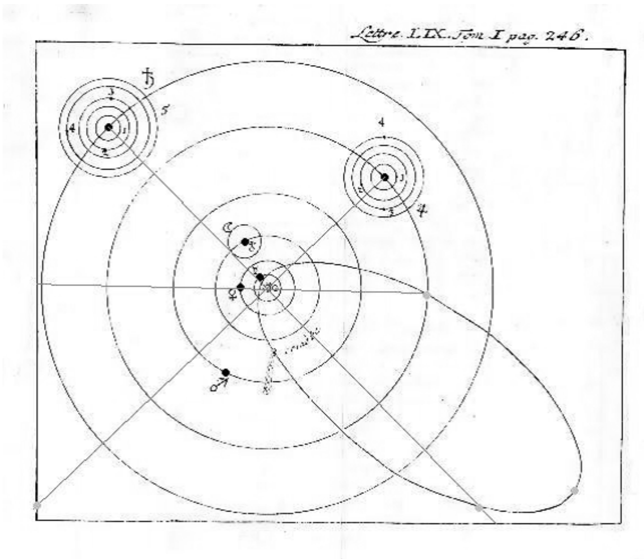
> **Expansion (P1)**, speed, sound, **music (P2)**, air, atmospheric pressure, air gun, light, luminescence, transmission of light, luminous body, color, refraction, refraction of different colors, color of the sky, plane mirror, **uneven mirror**, **Focusing mirror (P3)**, Focus, Mystery of the eye, Gravity, Shape of the Earth, Gravitation of the moon, Universal gravitation, Mutual gravitation between celestial bodies, **Solar system (PABC)**, Small changes due to mutual gravitation, Rising and ebbing tides, Universal gravitation explanation, properties of objects, inertia, change, monads, properties of forces, forces of other kinds.

The solutions to the first half of the problems are explained in [8]. In the second half of the problems, since E352 compared the zeta function to the sun and the moon, it would be natural to solution how other celestial bodies in the solar system can be compared to the zeta function. We infer that the solution is the irregular prime number of the solution to the zeta value problem PABC . The basis for this is the diagram of the solar system in "Letters to the Princess" on the next page. If we compare it with the diagram of irregular prime numbers below, we will notice that the directions of the planets and the irregular prime numbers almost correspond in order. Furthermore, from letters around the 103rd in ' 'Letters to the Princess", we can also associate 103 with one of many comets.

We believe that it would be difficult to find another diagram similar to Euler's solar system. Also, we realized that this book was the solution after we had solved the problems in the "Introduction", and we did not derive the solution based on the contents of the book.

Approximate values repeatedly displayed in "Introduction"

The approximation of $\log_{10} 2 = 0.3010300$ was repeatedly displayed because of the solution to PABC above: "7 irregular primes = 3 inner planets of the solar system + 1 comet + 3 outer planets" and the solution to P3. One possible reason is that it represents "seven bridges (3+1+3)". Please pay attention to the position of the comet in Euler's diagram of the solar system on the next page, and the depiction of bridge e in the diagram of the seven bridges.



Euler's solar system

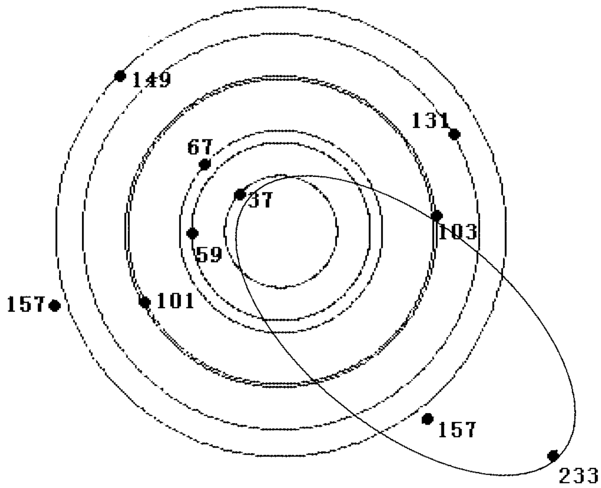
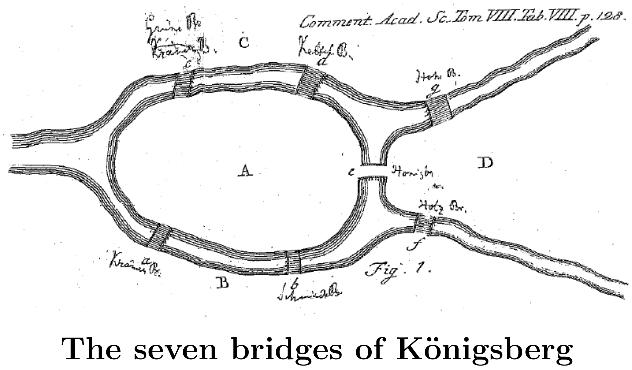


Diagram of irregular prime numbers



The seven bridges of Königsberg

It is assumed that the solutions to P1, P2, and P3 were also given in the "Letters to the Princess", as explained in [8]. In this way, we believe that many strange descriptions can be explained by this hypothesis. Rather than explaining the many strange descriptions and errors in terms of a "chain of coincidences", we think it would be easier to explain them in terms of Euler's "consistent intention".

Finally, we will list a method for disproving it. This is because the value of a hypothesis is determined by the existence of a method to disprove it.

A. Explain the mistake in the numerical value in another way.
B. Indicates that there are other similar numerical data.

Generally, if there is an error in a large number of errors, we can guess the reason why they are generated. There are 28 errors in the last digit of the 31 numbers in P2. If they are just calculation errors, we should be able to guess the reason. If it is difficult to explain the reason, we should present a similar list where the reason for its creation cannot be explained. But will it be possible to find a list of numbers in which the errors in nearly 30 numbers coincidentally sound like a song?

In addition, B can refute any fake codes in which characters are arbitrarily extracted from a large number of characters. All we have to do is show that such false codes exist at a certain rate in any text. However, we believe that it is extremely difficult to find an abnormal list like one in "Introduction".

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