第２４回仙台広島整数論集会 アブストラクト

７月８日（火）

10:00 – 10:50　倉本 篤希（九州大学）　/ Atsuki Kuramoto (Kyushu University)

On triple quadratic residue symbols in real quadratic fields

In this talk, I introduce triple quadratic residue symbols for certain primes in a real quadratic field k with narrow class number one. Our symbol may be regarded as a triple generalization of the quadratic residue symbol in k and also an extension of the Rédei symbol in the rationals for k. From the viewpoint of arithmetic topology, our symbol may be regarded as an arithmetic analogue of Milnor–Turaev's triple linking number of knots in a homology 3-sphere. For the construction, we determine the group presentation of the pro-2 maximal Galois group over k with restricted ramification, from which we derive our triple symbols by using mod 2 Magnus expansion. We then show that our triple symbol describes the decomposition law of a prime in a dihedral extension of degree 8 over k with restricted ramification, called Rédei type extension, and give concrete numerical examples. Finally, we give an interpretation of our symbols in terms of Massey products in Galois cohomology. I would like to explain how to construct Rédei type extensions, if possible.

11:05 – 11:55　柳原 亮祐（東北大学）　/ Ryosuke Yanagihara (Tohoku University)

捻れ Fermat 商曲線のルートナンバーと Fleck 数

Root numbers for twisted Fermat quotient curves and Fleck numbers

Let ℓ be an odd prime, $N \geq 1$ be an integer, and $\delta \geq 1$ be a ℓ^N-th power free integer such that $\mathrm{ord}_\ell(\delta) = 0$ or $\ell \nmid \mathrm{ord}_\ell(\delta)$. In this talk, we give an explicit formula for the root number of the Hecke character associated with a certain quotient curve of the twisted Fermat curve $X^{\ell^N} + Y^{\ell^N} = \delta$. This result gives a generalization of Stoll (2002) and Shu (2021). Through this calculation, we also have shown that the Fleck number which has been studied in detail in combinatorics, appears in the certain value of the Hilbert symbol.

13:30 – 14:20　軽部 尚香（大阪大学）　/ Naoka Karube (The University of Osaka)

The moduli space of dormant opers on elliptic curves

An oper is a fundamental object in the geometric Langlands correspondence, and in positive characteristic, it appeared naturally in the context of p-adic Teichmüller theory. In this talk, we focus on the global structure of the moduli spaces of dormant opers and dormant Miura opers on elliptic curves. We will show that these moduli spaces are connected DM stacks that are moreover finite and generically étale over the moduli space of elliptic curves, and we provide an explicit description of the étale locus. Furthermore, we introduce two extensions of the theory, one concerning dormant PGL_n-opers in prime-power characteristics and the other involving finite-level structures, and we discuss how these two aspects are related. This talk is based on the results presented in arXiv:2504.00418, which is a joint work with Yasuhiro Wakabayashi.

14:35 – 15:25　中田 裕貴（京都大学）　/ Yuki Nakata (Kyoto University)

代数的トーラスの被覆群の局所ラングランズ対応におけるパケットの S 群

Local Langlands correspondence for covering groups of algebraic tori, and the groups S parametrizing the packets

We compute the packets, precisely the groups S parametrizing them, in local Langlands correspondence for Brylinski–Deligne covering groups of algebraic tori, under some assumption on ramification. Especially, this work generalizes M. H. Weissman's result on covering groups of tori that split over an unramified extension of the base field.

15:40 – 16:30　深山 拓郎（東京大学）　/ Takuro Fukayama (The University of Tokyo)

The number of cuspidal representations over a function field and its behavior under base changes

Let X be a smooth projective curve over a finite field $\mathbb{F}_q$ and k be its function field. For a connected reductive group G over k, the structure of a cuspidal representation π of G is controlled by its local components π_v. We are interested in the number of cuspidal representations whose local components satisfy a given condition. Such a number should be computed by the sum of orbital integrals of G assuming Arthur's trace formula for function fields, and each orbital integral is related to an L-function of a motive of a reductive group which was introduced by B. H. Gross. I will explain the expression of the sum of the L-functions and its behavior under the base change from $\mathbb{F}_q$ to $\mathbb{F}_{q^m}$ for some classical groups.

16:45 – 17:35　藤井 天守（東京大学）　/ Amoru Fujii (The University of Tokyo)

Parametrization of supercuspidal representations of depth zero for some simple adjoint groups

We construct a surjective map from the set of conjugacy classes of depth-zero cuspidal enhanced L-parameters to that of isomorphism classes of depth-zero supercuspidal representations for simple adjoint groups, and check the bijectivity in various cases. We also prove that the Hiraga–Ichino–Ikeda conjecture on the formal degree of essentially square-integrable representations holds for this parametrization if it is bijective. We obtain these results by combining Feng–Opdam–Solleveld's local Langlands correspondence of unipotent supercuspidal representations and Lusztig's Jordan decomposition of irreducible characters of finite reductive groups.

7月9日（水）

9:30 – 10:20　水野 宏亮（名古屋大学）　/ Kosuke Mizuno (Nagoya University)

Graph-theoretic analogues of Brauer–Kuroda relations

We introduce the relations among the complexities of intermediate graphs of a Galois cover Y/X. Here, the complexities refer to the number of spanning trees of graphs. Hammer, Mattman, Sands, and Vallières prove that for a $(\mathbb{Z}/2\mathbb{Z})^m$-cover Y/X, the complexity of Y can be expressed in terms of the complexities of the intermediate graphs. We present a generalization of this result to arbitrary finite Galois covers. Our formulas are graph-theoretic analogues of Kuroda's formula and the Brauer–Kuroda relations in algebraic number theory. Furthermore, we examine the case where the Galois group is cyclic.

10:35 – 11:25　室岡 亮祐（名古屋大学）　/ Ryosuke Murooka (Nagoya University)

Iwasawa theory for vertex-weighted graphs

Gonet and Vallières independently proved that for a $\mathbb{Z}_p$-tower of graphs, an Iwasawa-type formula holds for complexities (= the number of the spanning trees) of the layers of the tower. In this talk, I will deal with the Iwasawa theory for vertex-weighted graphs. Vertex-weighted graphs have two kinds of complexities, namely rooted complexities and non-rooted complexities, and so we have rooted-Iwasawa-type formulas and non-rooted-Iwasawa-type formulas. Finally, I will present the Kida's formula for vertex-weighted graphs with an additional formula for Iwasawa μ-invariants.

11:40 – 12:30　後藤 有輝（慶應義塾大学）/ Yuki Goto (Keio University)

On factorizations of elliptic Soulé characters associated to once-punctured elliptic curves with complex multiplication

The Soulé character arises from the Galois action on the maximal metabelian quotient of the pro-p fundamental group of $\mathbb{P}^1 \setminus \{0, 1, \infty\}$, and can be regarded as an étale analogue of the Riemann zeta values at positive odd integers. Its counterpart for the once-punctured elliptic curve is called the elliptic Soulé character. In this talk, we show that for CM elliptic curves, the elliptic Soulé character at diagonal indices can be expressed as the product of the Soulé character and certain generalized Bernoulli numbers. This result allows us to reduce each property of the elliptic Soulé character in this case to its analogue for the Soulé character and Bernoulli numbers. As applications, we derive CM-elliptic analogues of the link between the Soulé character and Vandiver's conjecture, and of the Coleman–Ihara formula. This talk is based on joint work with Shun Ishii (Keio University).

13:50 – 14:40　劉 元旻（東京大学）　/ Yuanmin Liu (The University of Tokyo)

p-adic weight spectral sequences of strictly semi-stable schemes over formal power series rings via arithmetic D-modules

Over the Laurent series field $k((t))$, Lazda–Pál defined the $\mathcal{E}^\dagger$-valued rigid cohomology and Caro constructed the theory of arithmetic $\mathcal{D}$-modules. These p-adic cohomology and its coefficients theory contain more information of varieties over $k((t))$ compared to Berthelot's classical theory of $\mathcal{E}$-valued rigid cohomology or arithmetic $\mathcal{D}$-modules. In this talk, I will introduce the construction of weight spectral sequence using the theory of arithmetic $\mathcal{D}$-modules, which is an example of how $\mathcal{E}^\dagger$-valued p-adic cohomology reflects the geometry of varieties over $k((t))$.

14:55 – 15:45　渡邉 敬人（東京大学）　/ Takato Watanabe (The University of Tokyo)

On p-adic Galois representations of monomial fields and p-adic differential modules on fake annuli

The fake annuli introduced by Kedlaya are certain one-dimensional subannuli of p-adic polyannuli with multiple derivations. They are related to monomial fields, which are generalizations of Laurent series fields over fields of characteristic p. We compare the arithmetic and differential Swan conductors of rank one p-adic Galois representations of monomial fields with finite local monodromy. We also introduce a p-adic counterpart of monomial fields and explain generalizations of classical results to this setting, such as the overconvergence of p-adic Galois representations, and Berger's construction of p-adic differential modules from de Rham ones.

16:00 – 16:50　井上 絢太郎（京都大学）　/ Kentaro Inoue (Kyoto University)

Log prismatic F-crystals and comparison isomorphisms

Bhatt and Scholze introduced the theory of prismatic cohomology, which plays a striking role in integral p-adic Hodge theory. This cohomology theory is equipped with coefficient objects, called prismatic F-crystals. Guo and Reinecke studied the cohomology of prismatic F-crystals and applied it to proving the relative version of the $\mathrm{C}_{\mathrm{crys}}$-conjecture with non-trivial coefficients. I will talk about a logarithmic variant of this story, based on my previous work and ongoing work with Teruhisa Koshikawa.

17:05 – 17:55　近藤 海士（京都大学）　/ Kaiji Kondo (Kyoto University)

A group-theoretic description of the kernels of absolute trace maps and the application to the relationship between Aut-intrinsic Hodge–Tate-ness and crystallineness

In the present talk, let $k/\mathbb{Q}_p$ be a finite extension of degree d, $\overline{k}$ an algebraic closure of k, and G_k the absolute Galois group determined by $\overline{k}$. We show that the kernel of the trace map for the extension $k/\mathbb{Q}_p$ can be recovered group-theoretically from G_k, and can be described in terms of the generators of G_k. By means of this group-theoretic description of the kernel of the trace map for $k/\mathbb{Q}_p$, together with classical techniques from p-adic representation theory, we show that if p is odd, d is even, and $k/\mathbb{Q}_p$ is a Galois extension, then there exist a $\mathbb{Q}_p$-vector space V of dimension d, a continuous representation ρ: $G_k \to \mathrm{Aut}_{\mathbb{Q}_p}(V)$ that is irreducible, abelian, and crystalline (hence also Hodge–Tate), and a group automorphism φ: $G_k \xrightarrow{\sim} G_k$ such that $\rho \circ \varphi$ is not Hodge–Tate.

7月10日（木）

9:30 – 10:20　安澤 拓真（名古屋大学）　/ Takumi Anzawa (Nagoya University)

A Lie algebra corresponding to generalized symmetric multiple zeta values

Multiple zeta values (MZVs) $\zeta(k_1, ..., k_r)$ are real numbers associated with tuples of positive integers $(k_1, ..., k_r)$. Let $\mathcal{Z}$ be the $\mathbb{Q}$-algebra generated by all MZVs. The study of linear relations among MZVs is a central topic, motivated by the investigation of the corresponding algebraic groups represented by (lifts of) $\mathcal{Z}/\zeta(2)\mathcal{Z}$. The Kaneko–Zagier conjecture and its generalization by Rosen are conjectures formulated within the structure of the quotient $\mathcal{Z}/\zeta(2)\mathcal{Z}$. The generalization of the Kaneko–Zagier conjecture discusses the linear relations among p-adic finite multiple zeta values and generalized symmetric multiple zeta values. In this talk, we discuss a Lie algebra associated with generalized symmetric multiple zeta values under some assumptions.

10:35 – 11:25　ヤダデン ハレフ（名古屋大学）　/ Khalef Yaddaden (Nagoya University)

The double shuffle Lie algebra of N-congruent multiple zeta values

There are two primary level N generalizations of Multiple Zeta Values (MZVs): N-cyclotomic MZVs, which are multiple polylogarithm values at Nth roots of unity and N-congruent MZVs, which are MZVs defined via summations whose indices satisfy congruence conditions modulo N. Yuan and Zhao have shown that the linear spaces generated by each type of values are isomorphic over the cyclotomic field $\mathbb{Q}(e^{\frac{i2\pi}{N}})$. Both generalizations satisfy double shuffle relations, and Racinet has constructed a double shuffle $\mathbb{Q}$-Lie algebra in the cyclotomic setting denoted by $\mathfrak{dmr}_0^N$. In this talk, we introduce the double shuffle $\mathbb{Q}$-Lie algebra $\mathfrak{dmr}_0^{[N]}$ associated to the congruent case, and show that it is isomorphic to $\mathfrak{dmr}_0^N$ as $\mathbb{Q}(e^{\frac{i2\pi}{N}})$-spaces. Furthermore, we prove that, as $\mathbb{Q}$-spaces, $\mathfrak{dmr}_0^{[N]}$ is isomorphic to the subspace of $\mathfrak{dmr}_0^N \otimes \mathbb{Q}(e^{\frac{i2\pi}{N}})$ of invariant elements under the action of $\mathrm{Gal}(\mathbb{Q}(e^{\frac{i2\pi}{N}})/\mathbb{Q})$ (Joint work with Hidekazu Furusho).

11:40 – 12:30　横溝 真紘（東北大学）　/ Mahiro Yokomizo (Tohoku University)

Multiple L-functions and modular iterated integral

It is well known that modular L-functions admit both series and integral representations. Manin focused on the integral representation and introduced iterated integrals of cusp forms. Subsequently, Choie and Ihara revealed the relationship between such iterated integrals and multiple modular L-functions. More recently, Brown proved that the iterated integrals of a broad class of functions, including modular forms, are in fact rational functions. In this talk, I will report on a generalization of the work of Choie and Ihara to arbitrary modular forms. If time permits, I will also mention the application to multiple zeta values.

13:50 – 14:40　富山 和樹（早稲田大学）　/ Kazuki Tomiyama (Waseda University)

Generalized modular equations and arithmetic properties of singular values of Hauptmoduln

Monstrous moonshine relates the representation theory of the monster group to distinguished modular functions, called Hauptmoduln (principal modular functions). In this talk, we study the CM values of Hauptmoduln whose Fourier coefficients are cyclotomic integers, and prove that these are algebraic integers, analogous to the singular moduli of the elliptic modular j-function. The main tool for our proof is generalized modular equations for Hauptmoduln, introduced by the work of Cummins–Gannon.

14:55 – 15:45　藤吉 裕輔（九州大学）　/ Yusuke Fujiyoshi (Kyushu University)

On the finiteness of prime trees and their relation to modular forms

Let $\mathcal{M}_k(N, \mathbf{1}_N)$ denote the space of modular forms of level N, weight k, and trivial character. In 2018, Dickson and Neururer demonstrated that if $N = p^a q^b N'$, where p and q are distinct primes with $a, b \geq 2$, and N' is square-free and coprime to both p and q, then $\mathcal{M}_k(N, \mathbf{1}_N)$ decomposes as the sum of the Eisenstein subspace and the subspace spanned by products of two Eisenstein series. In our work, we extend their result to the case where N has up to six prime divisors p such that $p^2 \mid N$. By analyzing the underlying algebraic structure, we provide a complete classification in this range. Furthermore, for any fixed number of such prime divisors, we show that the Dickson–Neururer decomposition holds for all but finitely many levels N.

16:00 – 16:50　武田 暢輝（京都大学）　/ Nobuki Takeda (Kyoto University)

Differential operators on hermitian modular forms

It is well known that a modular form can be sent to some product of (vector-valued) modular forms with higher weights using differential operators. This is related to harmonic polynomials and has been studied in depth by Ibukiyama and others in the case of Siegel modular forms. In this talk, I will explain what happens in the case of hermitian modular forms on $\mathrm{U}_{n,n}$. In the first half of the talk, I will give an explanation from the viewpoint of representation theory. In the second half, I will explain the explicit construction method. If there is time, its applications will also be explained.

17:05 – 17:55　志賀 明日香（東北大学）　/ Asuka Shiga (Tohoku University)

BSD 不変量を共有する同型でない楕円曲線の組の無限族について

Infinitely many pairs of non-isomorphic elliptic curves sharing the same BSD invariants

The BSD conjecture for abelian varieties predicts that the leading coefficient in the expansion of the L-function at $s = 1$ can be expressed in terms of BSD invariants (Mordell–Weil group, regulator, real period, Tamagawa numbers, and the Tate–Shafarevich group). When all BSD invariants coincide, this condition imposes strong constraints on abelian varieties; however, Jamie Bell constructed an example of a non-isomorphic pair of 22-dimensional abelian varieties for which the BSD invariants (excluding the real period) and the Selmer groups and Tate modules coincide (over any number field). In this talk, I will discuss my results on infinite families of pairs of non-isomorphic elliptic curves that share BSD invariants and Kodaira symbols. Controlling the Tate–Shafarevich groups of two elliptic curves by density theorem for twist of Selmer groups is a key point.

7月11日（金）

9:30 – 10:20　伊藤 遥来（名古屋大学）　/ Haruki Ito (Nagoya University)

Semi-integral points of bounded height on equivariant compactifications of vector group

Let X be an algebraic variety over a number field F and let $H_{\mathcal{L}}$ denote the counting function on the set $X(F)$ of rational points with respect to an adelically metrized line bundle $\mathcal{L} = (L, |\cdot|)$ on X. Manin's conjecture concerns an asymptotic formula for the counting function $N(U, \mathcal{L}, B) = \#\{P \in U \mid H_{\mathcal{L}}(P) \leq B\}$ for a suitable subset U of $X(F)$. This conjecture was proposed by Y. Manin and his collaborators in the late 1980s. In this talk, we will prove Manin's conjecture for Darmon points on compactifications of vector groups, following ideas from the work of M. Pieropan, A. Smeets, S. Tanimoto, and A. V. Alvarado.

10:35 – 11:25　田代 大尭（九州大学）　/ Hirotaka Tashiro (Kyushu University)

３次元トポロジーにおけるハッセのノルム定理と種の理論

Hasse norm theorem and genus theory in 3-dimensional topology

Based on the analogies of arithmetic topology, I present a topological analogue of the genus theory for integral homology 3-spheres. As is well known, genus theory was initiated to be studied by Gauss and since then many mathematicians has investigated. By arithmetic topology which bridges between number theory and 3-dimensional topology, its topological analogue was studied by Reznikov, Morishita, and Ueki. In this talk, I will present a topological genus theory for cyclic covers of integral homology 3-spheres in an idelic manner. For this, I employ Niibo–Ueki's idelic class field theory for 3-manifolds, my previous work on a topological Hasse norm theorem and an analogue of Hilbert 90 theorem. Our proof is perfectly and beautifully parallel to Iyanaga–Tamagawa's proof for the arithmetic genus theory.

11:40 – 12:30　北島 雅也（名古屋大学）　/ Masaya Kitajima (Nagoya University)

Generalization of Hardy's identity for lattice points of astroid-type closed curves via Erdélyi–Kober operator

Hardy's identity is a series representation for the error term of lattice points of a circle, derived by G.H. Hardy in 1915. This series is composed of the Bessel function of order 1 and a certain number-theoretic function, and by appropriately transforming them, we have obtained the counterpart for the generalized circle p-circle when $2/p$ is a natural number. In this talk, we will emphasize that a derivative formula, that is, a property of generalized Bessel functions, is the key to the derivation, and that it is closely related to the fractional derivative, in particular, to the Erdélyi–Kober operator.

13:50 – 14:40　伏見 宗紘（京都大学）　/ Tokihiro Fushimi (Kyoto University)

正標数代数曲線の基本群の特殊化射の非同型性

Non-isomorphic specialization morphisms of fundamental groups of algebraic curves in positive characteristic

"Anabelian geometry over an algebraically closed field (of positive characteristic)" is a field motivated by the question of how much the original hyperbolic curve can be 'reconstructed' from its étale fundamental group. The above 'reconstruction' does not hold at all in the case of characteristic 0. However, the situation is quite different in the case of positive characteristic p, and this result was obtained by Akio Tamagawa in the early 2000s. In this talk, after explaining this field, I will introduce Raynaud's theta divisor, which is a tool used in this field. Furthermore, I will present my recent results regarding specialization morphisms between the fundamental groups of geometric curves corresponding to each point on the coarse moduli space $M_{g,[n],\mathbb{F}_p}$.

14:55 – 15:45　玄 承賢（東京科学大学）　/ Seung-Hyeon Hyeon (Institute of Science Tokyo)

The m-step solvable anabelian geometry of mixed-characteristic local fields

Let $K_\circ$ (resp. $K_\bullet$) be a mixed-characteristic local field, and $G_{K_\circ}$ (resp. $G_{K_\bullet}$) its absolute Galois group. Mochizuki (1997) has shown that every isomorphism between $G_{K_\circ}$ and $G_{K_\bullet}$ that preserves the ramification filtration is induced by some field isomorphism between $K_\circ$ and $K_\bullet$. In particular, if there exists an isomorphism between $G_{K_\circ}$ and $G_{K_\bullet}$ that respects the ramification filtration, then there also exists a field isomorphism between $K_\circ$ and $K_\bullet$. In this talk, I would like to introduce a recent result that can be considered as an "m-step solvable version" of that of Mochizuki, alongside the *mono-anabelian* aspect of the theory.